A PRIMER

OF

ORTHOGRAPHIC

PROJECTION

MAJOR G. T. PLUNKETT, R.E.

A PRIMER

OF

ORTHOGRAPHIC PROJECTION;

OR,

ELEMENTARY PRACTICAL SOLID GEOMETRY CLEARLY EXPLAINED,

WITH

NUMEROUS PROBLEMS AND EXERCISES.

SPECIALLY ADAPTED FOR SCIENCE AND ART CLASSES, AND FOR THE USE OF STUDENTS WHO HAVE NOT THE AID OF A TEACHER.

BY MAJOR G. T. PLUNKETT,
ROYAL ENGINEERS.

London:
SAMPSON LOW, MARSTON, SEARLE, & RIVINGTON,
CROWN BUILDINGS, 188, FLEET STREET.
1885.

PREFACE.

It is needless to enlarge upon the great importance of the subject of which the elementary truths are taught in the following pages, whether as an excellent mental exercise or as a necessary foundation for theoretical training in any of the mechanical arts and sciences. Many thousands of pupils, preparing for a variety of occupations, study it in schools of every class or in their leisure time throughout the country. Could all these learners find teachers within reach, and had every teacher ample time to devote to individual instruction, some of the larger works published on this subject would perhaps suffice as class-books, but this is far from being the case.

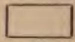
Most pupils have to learn in large classes, where the teacher can afford but little time for explaining difficulties as they are met with by individuals; many learn, and no doubt many more wish to learn, in places where qualified teachers are not to be obtained, and the learner must rely upon his books alone. To assist these, and to facilitate the work of teaching, is the object of the following primer, and the author believes that no student who studies it diligently and works the examples will have any difficulty in satisfying the examiners of the Science and Art Department. Candidates for a second-grade certificate in art need only go as far as the end of the fourth chapter; science students must work through the whole of the primer, and should then have no difficulty in passing the examination in the elementary stage, while they will be well prepared for taking up the more advanced works on the subject.

ORTHOGRAPHIC PROJECTION.

CHAPTER I.

PRELIMINARY OBSERVATIONS.

THE branch of geometrical drawing taught in the following pages is one by which we make drawings of various objects (for instance, of buildings or parts of buildings) when it is intended that workmen shall use these drawings, and be able to take from them the lengths, breadths, and heights of the buildings or other objects represented. It is therefore very different from the mode of drawing adopted for ordinary pictures; in these the artist works according to the rules of perspective, that is as objects naturally appear in our eyes, those which are nearest to us looking larger than those which are farther off. For instance, look at any picture in which there is a regular row of houses represented with one end of the row farther off than the other, you will see that the house nearest to the front of the picture is drawn the largest, and each succeeding house smaller and smaller to the most distant, which (if the actual houses are of one size) will appear the smallest of them. Now it would be useless to give such a picture to a builder, and to expect him to take from it the widths and other dimensions of the houses in the row; he must be given drawings made by the rules of "orthographic projection," or, as they are usually called, a "plan," and one or more "elevations" of the row, and from these he will be able to see the intended length, breadth, and height of each house.

To understand this more fully, take a small box, such as a box of drawing-instruments, and place it upon a sheet of paper, draw four lines on the paper with your pencil, closely round the bottom of the box, forming a rectangle thus  and remove the box. You have now drawn what is called a "plan" of the box, and from it you can evidently ascertain the length and the width of the box, but you cannot tell what is its height. Now place a sheet of paper upright against a wall, with one hand hold the box, so that one side of it is pressed against the paper, and with the other again draw four lines upon the paper close to the box, that is to say, along the top of it, down the two ends, and along the bottom; remove the box, and you have upon the paper a "side elevation" of it, from which you can measure the length and the height of the box, but you cannot tell what is its width. Lastly, still keeping the paper on the wall, hold one end of the box against it, and draw closely round the end as before with your pencil; you will thus obtain a third rectangle, the same size as the end of the box, or what is called an "end elevation," and from this you can measure the width of the box and its height.

Suppose your first drawing, which is of course the same size as the bottom of the box, is eight inches long and five inches wide, and that your second, which is the same size as the side of the box, is eight inches long and two inches high, and that your third, which is the same as the end of the box, is five inches wide and two inches high; if you give these drawings to a joiner, he can make another box of exactly the same dimensions, for from the first he will see that the box is eight inches long and five inches wide, and from the second he will know that the box is eight inches long and two inches high, and from the third that it is five inches wide and two inches high. It would not, in this case, be necessary to give him all three drawings; from any two of them, from the first and second, or from the first and third, or from

the second and third, he could find the three dimensions required; that is to say, the length, width, and height of the box.

Now it is evident that, generally speaking, objects which one may have to draw, such as buildings or parts of buildings, furniture, ships, &c., cannot be placed upon the drawing-paper or held up against a wall, in order to make plans and elevations of them. To learn the rules by which this is done, it is necessary first to learn how to find the plans and elevations of points in various positions, then of lines, and then of simple solids. When this is thoroughly understood, no difficulty will be found in drawing the projections of more complicated objects, and instead of making the drawing the same size as the object, it may have to be made one-fourth, one-tenth, one-hundredth, or some other fraction of the real size.

As a first step it is necessary to understand clearly what are vertical and horizontal lines and planes. If you tie a weight to a string and let it hang, the string is vertical, for it points towards the centre of the earth, and a *vertical line is one which is drawn towards the centre of the earth;* and if you place a board or a sheet of paper with its surface touching the side of the string, then this surface of the board or paper is a *vertical plane*, for a *vertical plane* is a plane in which vertical lines can be drawn. If several such lines are drawn, it is evident that they are not absolutely parallel to each other, for if produced they would meet at the centre of the earth; but this is so far off, that for all practical purposes we may take them as parallel. The smooth face of an ordinary wall is an example of a vertical plane.

A horizontal line is a line which is at right angles to a vertical line. For instance, again hold the string with a weight at the lower end of it against an upright wall (which is the same thing as holding a sheet of paper so that its surface may touch the string), and draw a line down where the string touches the wall (or paper), then this is a vertical line. Now draw a line cross-

ing this vertical line at right angles, this line will be a *horizontal line*, or what is usually called *level.*

A *horizontal plane* is one in which any line which can be drawn is horizontal. The smooth surface of a table or a floor should be a horizontal plane, and so is the surface of still water in a basin or other vessel.

The surface of the drawing-paper on which we work is of course a plane, and if looked at edgewise, by placing the eye level with the paper at one side of it, then the surface of the paper or plane will appear as a straight line.

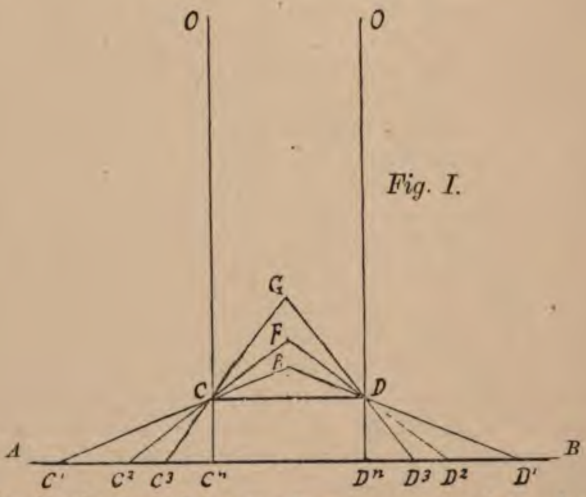


Fig. I.

Now let the line AB in Fig. I. represent the plane of your drawing-paper when laid upon a table and seen edgewise, and taking any object such as a pencil or penholder, hold it *horizontally* (that is *level*) about a foot above your drawing-paper,[1] and let the two ends of it be called C and D.

[1] It will be better to get some one else to hold the pencil or penholder while you place your eye in position and mark the points on the paper.

If you place your eye a few inches above the pencil as at E, and mark upon the paper the two points which the two ends C and D seem to cover, you will see that the object CD covers on the paper a certain length C^1D^1, which is much more than its actual length. Now raise your eye rather higher as to F, and you will see that the object covers rather less space than it did before, for if you mark the points covered by the two ends they will be as C^2D^2 inside C^1 and D^1; that is to say, the projection C^2D^2 will be nearer to the actual length CD than C^1D^1 was.

Again raise your eye still further above the object as to G, and mark the points covered by C and D, you will find they are as C^3 and D^3 within C^2 and D^2, that is to say, the projection C^3D^3 is still nearer to the true length of CD than C^2D^2 was; and the farther you remove your eye from the object, the more nearly will the length of the projection of it (that is to say the length it covers upon the paper) approach to the true length of the object CD. You will also note that the angle between the lines drawn from the eye in its different positions (first ECC^1 and EDD^1, secondly FCC^2 and FDD^2, thirdly GCC^3 and GDD^3, and so on) continually diminishes, the angle at F being less than the angle at E, and the angle at G less than the angle at F. At last, if you imagine the eye removed to an infinite distance from the object, this angle may be regarded as nothing, and the lines may be taken as parallel as OCC^n and ODD^n, and in this case the projection C^nD^n becomes the same length as that of the object CD itself. This C^nD^n is called an "*orthographic projection*" of CD upon the plane AB, and the lines OGC^n and ODD^n are called "*projectors*," and as they are vertical lines (for they are at right angles to the horizontal plane AB) they are called "*vertical projectors*," and the projection C^nD^n being on a horizontal plane is called a "*horizontal projection*" or "*plan*."

Now, in the above experiment, we have supposed the drawing paper to be lying upon a table, and therefore to represent a

horizontal plane ; but it would equally hold good if the paper had been fixed against an upright wall so as to represent a *vertical plane.* In this case the two lines OCC^n and ODD^n instead of being *vertical* become "*horizontal projectors,*" and the line C^nD^n becomes a "*vertical projection*" or "*elevation.*" It will be seen that vertical projectors must always give a *horizontal projection* or *plan,* while *horizontal projectors* must give a *vertical projection* or *elevation.*

The "*trace*" of a line on any plane is the point in which the line meets the plane. Thus C^n is the trace of the projector OCC^n, and D^n is the trace of ODD^n upon the plane AB. Traces which are in a horizontal plane are called *horizontal,* and those in a vertical plane are called *vertical.*

The plan of a point, therefore, is the *horizontal trace* of a *vertical* line or *projector* passing through that point, and the elevation of a point is the *vertical trace* of a *horizontal projector* passing through the point.

The trace of a *plane* is the line in which it cuts another plane. Take again a sheet of paper lying upon a table to represent a horizontal plane, and hold an object, such as a pen or pencil, to represent as before a line CD a short distance above it ; then hold another piece of paper or cardboard upright against the pen or pencil with its lower edge resting on the drawing-paper in the line C^nD^n ; then the line C^nD^n, which is the plan of CD, is the horizontal trace of a vertical plane passing through CD.

The plan of a line, therefore, may be defined as *the horizontal trace of a vertical plane passing through that line.*

It is absolutely necessary that the student should comprehend fully the above preliminary remarks and the following observations and problems on the projections of points before attempting to proceed further. It will be useless for a teacher 'ry to explain them by means of figures upon a blackboard or for the learner to trust only to the figures given in

these pages. He must use two pieces of paper, cardboard, or wood to represent the vertical and horizontal planes, and wires or pins to represent projectors, and then the student will have no difficulty in understanding clearly the principles upon which this science is based.

Let the student then take two pieces of paper or cardboard as ABCD and CDEF in Fig II., and place them with the

Fig. II.

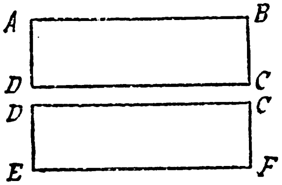
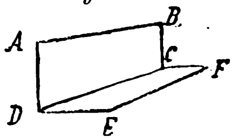


Fig. III.



two edges marked CD touching each other, but letting one piece (say CDEF) lie flat upon the table, while the other, ABCD, is stood upright by being propped with books or placed against the wall, as in Fig. III., to represent a horizontal and a vertical plane. Let him turn these two boards or sheets of paper until one end only is opposite to his eye, that is to say, so that he may see the ends AD and DE only, as in Fig. IV. Then let him take any small object to represent a point G (a pea or morsel of india-rubber on the end of a wire or on a pen will do) and hold it somewhere between the two planes. Now, if the wire be held vertically with the lower end resting upon the horizontal plane at g, and with the object (which may be supposed to represent a mathematical point) at the top of it G, the wire Gg will represent a vertical projector, and g will be its horizontal trace, that is to say, g will be the *plan* of the point G. Similarly, if the wire be held horizontally with one end touching the vertical plane in g' and the object G at the other end, the wire from g' to G will represent a horizontal projector, and g'

will be its vertical trace, that is to say, g' will be the elevation of G.

Now it is easy to represent all this on paper, as in Fig. IV., so long as the planes are only seen edgewise from one end; but when seen from any other position they cannot be drawn on the page of a book, as they should be, at right angles to each other. The course adopted, therefore, is to suppose the horizontal plane CDEF to lie flat upon the paper, and to imagine the vertical plane ABCD to revolve or be turned backwards upon the line CD until it also lies flat upon the paper. The student must, as above mentioned, take two pieces of paper, card, or wood, let one lie flat upon the table to represent the horizontal plane CDEF, and hold the other upright to represent the vertical plane ABCD, and then he must turn this vertical plane backwards, keeping the edge CD close against the edge of the horizontal plane, until it also lies flat upon the table as ABCD and CDEF on page 11; and he must turn the vertical plane ABCD up and down as required to understand the following explanations.

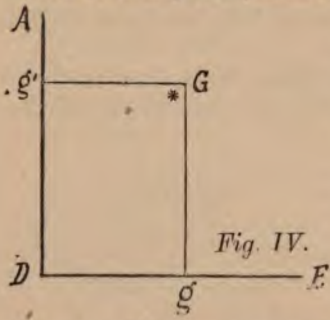
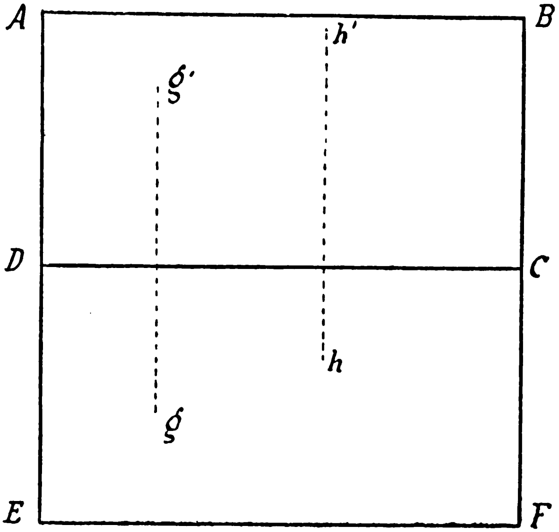


Fig. IV.

Now, if the object G was originally one inch above the horizontal plane, that is, if the wire, or penholder, or needle, or whatever made the vertical projector Gg was one inch long, it is evident that the elevation g' in Fig. IV. will be one inch above the point D where the two planes meet; and if another object H is $1''\cdot25$ above the horizontal plane, the vertical projector Hh is $1''\cdot25$ long, and the elevation h' is $1''\cdot25$ above the point D.

When the vertical plane is turned back, as in figure on p. 11,

the elevation g' is evidently still $1''$ from CD, and the elevation h' is $1''\cdot25$ from CD.



Again, if the object G was $\cdot75''$ in front of the vertical plane, that is if the horizontal projector Gg' was $\cdot75''$ long, it is evident that the plan g in Fig. IV. is $\cdot75''$ from D, and it will be $\cdot75''$ from CD in above figure. Similarly, if H was $\cdot5''$ in front of the vertical plane, the plan h will be $\cdot5''$ from D in Fig. IV. and from CD, in above figure.

The line CD, in which the vertical meets the horizontal plane, is called the "*ground-line*," and it is evident that the lines between g and g' or h and h' in above figure will always be at right angles to CD; hence we derive the following very important rule:—

"The plan and elevation of any point lie always in one straight line, which is at right angles to the ground-line."

CHAPTER II.

PROJECTIONS OF POINTS.

So far we have considered that the vertical plane and the horizontal plane end at the line CD where they meet; but as every plane may be extended, or supposed to be extended, to any length and any breadth, we may suppose that the vertical plane instead of only meeting *passes through* the horizontal in the line CD.

The student may represent this by taking two cards, and cutting a slit as far as the centre of each from opposite ends, and then

fitting the two together in the form of a cross, as shown in Fig. V. Now turn this cross so as to be seen edgewise, as in Fig. VI., where ADM is the edge of the vertical plane, and ODE the edge of the horizontal plane, D being the end of the line CD where the planes cross. The angle formed by two planes is called a dihedral angle, the angle between the upper part of the vertical plane ABCD and the front part of the horizontal plane CDEF in Fig. III. or Fig. IV. is called the first dihedral angle, and this is the same as the angle ADE in Fig. VI.

Similarly, the angle behind the vertical plane and above the horizontal plane, which is represented in Fig. VI. by the angle

ADO, is called the second dihedral angle ; the angle behind the vertical plane and below the horizontal, ODM, is called the third, and the angle before the vertical and below the horizontal plane, EDM, is called the fourth dihedral angle.

We have seen in pages 10 and 11 that if an object G is in the first dihedral angle its plan is in CDEF, the front part of the horizontal plane, and its elevation g' is in ABCD the upper half of the vertical plane. It will be well to consider more at

Fig. VI.

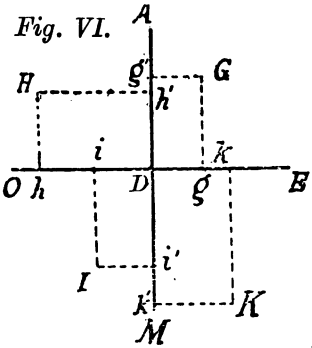
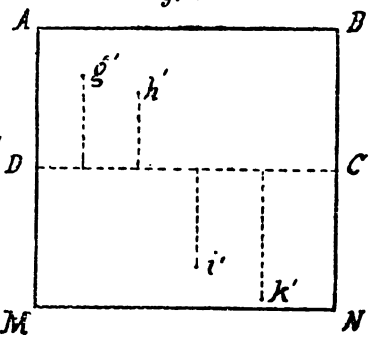


Fig. VII.



length where the plans and elevations of points in the different angles will fall.

Let Fig. VI. represent the two pieces of card joined together in the form of a cross, and let Fig. VII. represent the vertical plane, and Fig. VIII. the horizontal plane, when the cards are separated. (Note. In these figures the dimensions drawn, are reduced to a half or less of those given in the descriptions, in order to save space.)

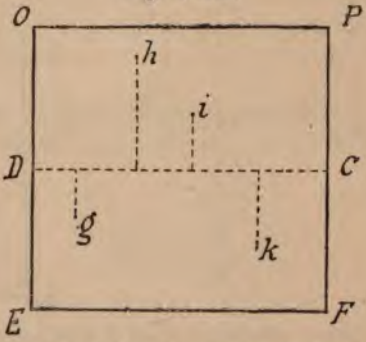
The point G in the first dihedral angle we will suppose at 1″ above the horizontal plane and 0″·75 in front of the vertical plane, then in Fig. VI. the elevation g' is 1″ above the point D and the plan g is 0″·75 from D towards E, and when the cards are separated g' will appear as in Fig. VII. at 1″ above

the ground-line CD, and g will appear as in Fig. VIII. at $0''\cdot75$ below CD.

Take another object, H in the second dihedral angle, say at $0''\cdot8$ above the horizontal and $1''\cdot2$ behind the vertical plane, then in Fig. VI. the elevation h' will be at $0''\cdot8$ above the point D and the plan h will be in DO at $1''\cdot2$ from D, and when the cards are separated h' will be as in Fig. VII. at $0''\cdot8$ above CD, and h will be as in Fig. VIII. at $1''\cdot2$ above CD.

If a third object I be in the third dihedral angle, say $0''\cdot6$ behind the vertical and $1''$ below the horizontal plane, the elevation i' will be in the lower part of the vertical plane as in Fig. VI. at $1''$ below D and the plan i in DO at $0''\cdot6$ from D, and in Fig. VII. i' will be $1''$ below CD, and in Fig. VIII. i will be $0''\cdot6$ above CD.

Fig. VIII.



Lastly, if K be in the fourth dihedral angle at $0''\cdot8$ in front of the vertical and $1''\cdot4$ below the horizontal plane, the elevation k' will be $1''\cdot4$ below D and the plan k $0''\cdot8$ in front of D in Fig. VI., and in Figs. VII. and VIII. the elevation will be $1''\cdot4$ below CD and the plan $0''\cdot8$ below CD, respectively.

Now putting the two cards together again as in Figs. V. and VI., turn the upper part of the vertical plane ABCD back upon the back half of the horizontal OPCD, so that it may coincide with it, the lower half of the vertical DMNC being turned forwards until it coincides with the front part of the horizontal plane CDEF; then supposing the cards to be actual mathematical planes without any thickness, the points which are in

OPCD will appear with those in ABCD, and the points in DMNC will appear in CDEF as in Fig. IX.

From these four examples we may deduce the following rules:—

1. If a point is in the first dihedral angle, its plan (when represented on a flat sheet of paper in the ordinary way) will be below and its elevation above the ground-line, as *g* and *g'* in Fig. IX.

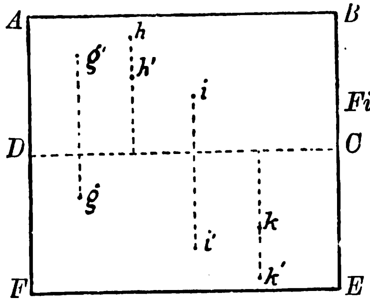


Fig. IX.

2. If a point is in the second dihedral angle, its plan and elevation will be both above the ground-line, as *h* and *h'*.

3. If a point is in the third dihedral angle, its plan will be above and its elevation below the ground-line, as *i* and *i'*.

4. If a point is in the fourth dihedral angle, the plan and elevation will both be below the ground-line, as *k* and *k'*.

The student should now have perfectly mastered the principles above explained, and they are further illustrated in the following simple problems.

In these, the ground-line is distinguished (as is the common practice) by the letters XY, the original points discussed are marked with the large Roman letters A, B, C, D, &c., their plans with the small italic letters *a*, *b*, *c*, *d*, &c., and the elevations with similar letters, with a dash ′ above, as *a'*, *b'*, *c'*, &c.

To economize space, H.P. and V.P. are used in place of the words horizontal plane and vertical plane respectively.

Problem I.

To determine the projections of the points A, B, C, D, E, F, and G, when in the following positions,—

The point A to be 1″·8 in front of the V.P. and 1″·6 above the H P.

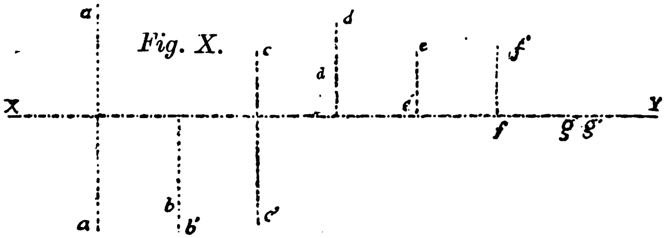


Fig. X.

The point B to be 1″·4 in front of the V.P. and 1″·8 below the H.P.

„ C „ 1″ behind the V.P. and 1″·7 below the H.P.

„ D „ 1″·5 behind the V.P. and 0″·5 above the H.P.

„ E „ 1″ behind the V.P. and in the H.P.

„ F „ in the V.P. and 0″·5 above the H.P.

„ G „ in both planes.

Take any line XY for a ground-line. Then as the point A is 1″·8 in front of the V.P., take for its plan a point *a* 1′·8 below XY, and, as "the plan and elevation of any point must always be in one straight line at right angles to the ground-line," through *a* draw a line at right angles to XY, and take on it a point *a*′ at 1″·6 above XY; this will be the elevation of A, which is in the first dihedral angle.

The point B is in the fourth angle, and as it is 1″·4 in front of the V.P., the plan *b* will be 1″·4 below XY, and a line being drawn at right angles to this the elevation will be in it at 1″·8 below XY.

The plan of C will be 1″ above XY, and the elevation 1″·7 below it.

The plan of D will be 1″·5 above XY, and the elevation 0″·5 above it.

The plan of E will be 1″ above XY, and as the point is in the H.P., its elevation will be in the H.P., that is to say, it will be in the line XY.

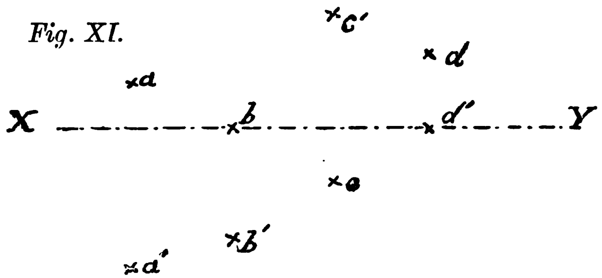
Similarly, as F is in the V.P., its plan will be in the V.P., that is to say, in XY, and its elevation will be 1″ above XY.

Lastly, as the point G is in both planes it must be in the line where they intersect, that is to say, in the ground-line, and the plan and elevation will coincide with the point itself.

Problem II.

Describe the position of the points ABCD, the projections of which are shown in the accompanying figure, which is drawn to a scale of one-half the actual dimensions.

Fig. XI.



This is the converse of the preceding problem.

By measurement with the compasses and a scale it is found that the plan a is 0″·25 above and the elevation a' 0″·75 below the ground-line, and as above mentioned, these represent respectively 0″·5 and 1″·5. From the plan, therefore, it is evident the point A is 0″·5 behind the V.P., and from the elevation that it is 1″·5 below the H.P.

b is in the ground-line, and b' is found by measurement to be $0''\cdot6$ below it; hence it appears that the point B is in the V.P. and $1''\cdot2$ below the H.P.

c is $0''\cdot3$ below, and c' $0''\cdot6$ above XY, hence the point C is $0''\cdot6$ in front of the V.P. and $1''\cdot2$ above the H.P.

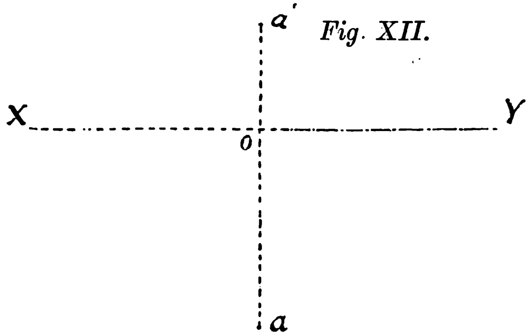
d is $0''\cdot4$ above, d' is in XY, whence it is known that the point D is $0''\cdot8$ behind the V.P., and is in the H.P.

PROBLEM III.

The projections of a point A being given, to find its actual distance from the ground-line.

Let a be the plan and a' the elevation of A, as shown in Fig. XII., and let the line which joins them meet the ground-

Fig. XII.



line XY in O; then we know that the point A must be above the H.P. a distance equal to O a', and that it is in front of the V.P. a distance equal to O a. Now suppose the two planes to be in their proper position with regard to each other, that is to say, *crossing each other* as in Figs. IV., VI., and to be turned so that both are seen edgewise as in Fig. XIII., and mark off O a' on the line which represents the V.P. and O a on the line which represents the H.P.; draw through a' a line parallel to the H.P., and through a a line parallel to the V.P., which will

represent the horizontal and vertical projectors respectively; then the point in which these lines intersect must be the

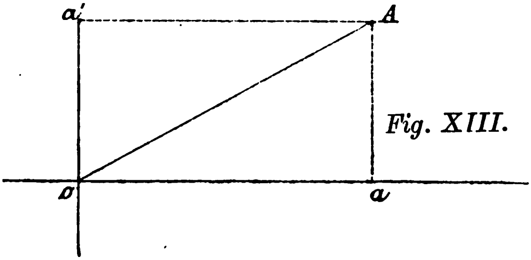


Fig. XIII.

point A, the plan of which is a and the elevation a', and if we join AO this will be the actual distance of the point A from ground-line. It is usual, in order to save time, to combine two or more such figures, as XII. and XIII., into one, as in Fig. XIV.

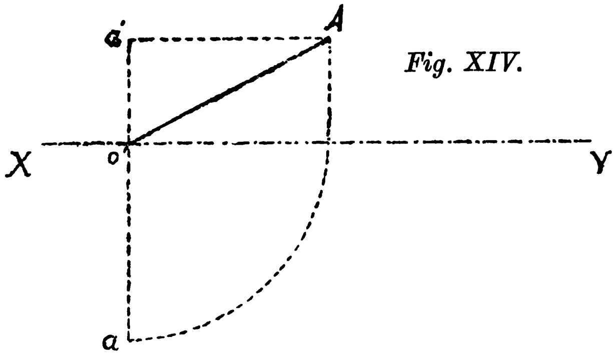


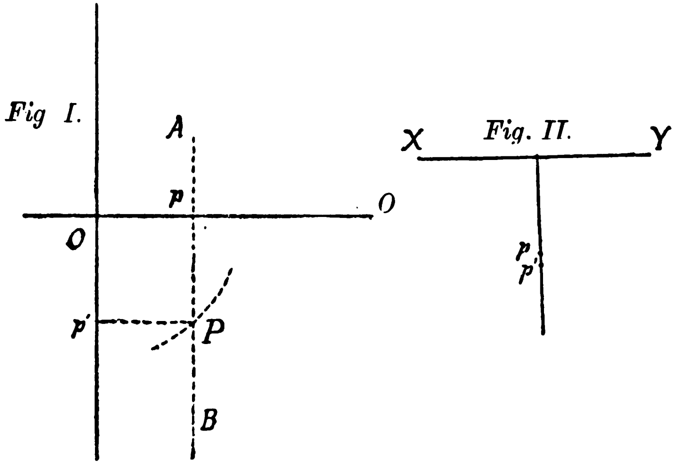
Fig. XIV.

PROBLEM IV.

A point P is 1″ in front of the vertical plane, it is below the horizontal plane, and its distance from the ground-line is 1″·5; determine its plan and elevation.

This is the converse of the preceding problem.

Draw two lines to represent the two planes seen edgewise as in Fig. I., draw a line AB parallel to the line which repre-



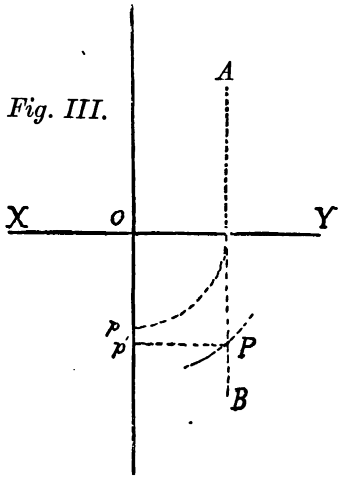
sents the V.P., and 1″ from it (on whichever side is taken to represent the front), then as the point P is 1″ in front of the V.P. it must be in this line; and as it is 1″·5 from the ground-line, which is represented in this figure by the point O where the lines representing the planes cross, from O with radius 1″·5 describe an arc cutting AB in P, which will be the point required. As AB will represent a vertical projector, the point p where it intersects the line of the H.P. will be the plan, and if through P a line is drawn parallel to the line of the H.P. meeting the line of the V.P. in p', this will represent the elevation of P.

In Fig. II. the plan and elevation are shown in the usual way, in a line drawn in right angles to XY, the distances of p and p' from XY being made equal to Op and Op' in Fig. I.

Fig. III. shows how this would usually be worked out in one figure; the line of the H.P. is used as the ground-line XY, and the point p is found by describing, with O as centre, an

arc from the point where the line AB cuts the line of the H.P. to meet the line of the V.P. in p.

Fig. III.



EXAMPLES.

1. Show the plans and elevations of the following points:—
   A. 3″ above the H.P. and 1″·8 before the V.P.
   B. 2″ above the H.P. and 2″·2 behind the V.P.
   C. 1″·5 below the H.P. and 0″·4 before the V.P.
   D. 0″·75 below the H.P. and 1·3 behind the V.P.
   E. 2″·3 below the H.P. and in the V.P.
   F. In the H.P. and 1″·6 before the V.P.
   G. In both planes.

2. A point A is 1″·8 from XY and 1″·2 below the H.P. and it is before the V.P.; find its plan and elevation.

3. A plan a is 1″·3 below XY and a' the elevation of the same point is 0″·8 above XY; what is the true distance of the point A from XY?

4. The plan of a point is 2″ above XY and the elevation is in XY; what is the actual distance of the point from the ground-line?

CHAPTER III.

PROJECTIONS OF LINES.

As points may be in any of the four dihedral angles, so also a line may be entirely in any one of the angles, or it may be in two or in three of the angles by passing through one or both of the planes.

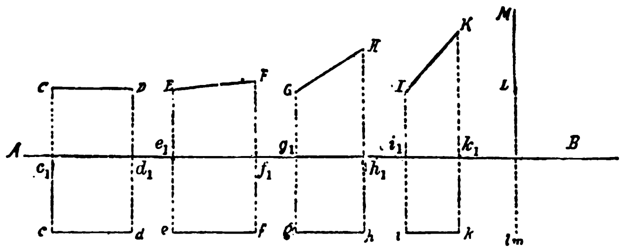
The projection of a line consists of the projections of all the points in it from one end of the line to the other; therefore if a line is straight it is evident that the projections of it must be straight lines.

As regards the position of a line, it may be parallel to both the planes, in which case it will evidently be parallel to the ground-line in which they meet, or it may be parallel to the horizontal plane and inclined to the vertical, or parallel to the vertical and inclined to the horizontal, cr it may be inclined to both the planes.

Taking as before two cards crossed at right angles to represent the two planes, hold a pencil parallel to the H.P.; it is evident that the plan of it will be exactly the same length as the pencil. Next raise one end of it higher than the other so that it is inclined to the horizontal, then its plan will be a line less than the length of the pencil, and as we increase the inclination the length of the plan will diminish, until at last, when the pencil becomes vertical, its plan becomes a point only—as illustrated in the figures below. Again, when the pencil is vertical it is parallel to the V.P., and its elevation is evidently equal in length to the pencil itself.

If AB represent a H.P. seen edgewise, then CD in the first figure on the left will represent the pencil (supposed to be a *line*) when horizontal, and c_1d_1 the portion of the line AB intercepted between two vertical projectors from C and D will be the same length as CD.

Now c_1d_1 is the plan when the H.P. is looked at edgewise, the eye being placed level with the plane, and if the H.P. is looked down upon as a sheet of drawing-paper usually is when in use the plan will appear as *cd*, which will of course be of the same length as CD; *c* is the plan of the point C and *d* is the plan of the point D, and every point in the line *cd* is the plan of a corresponding point in the line CD.



Of course the distance of *cd* from AB has nothing to do with the question, but if AB were taken to be the usual ground-line XY, the distance from it to *cd* would be the distance of CD from the V.P.

In the second figure EF will represent the pencil when held, not horizontal, but slightly inclined to the H.P., then e_1f_1 and *ef* will evidently be shorter than EF; and in the next figure, where GH is more inclined to the H.P., g_1h_1 and *gh* will be shorter than e_1f_1 and *ef*.

In the fourth figure, where IK represents the line or pencil still more inclined to the H.P. than in the third, i_1k_1 and *ik* will be still shorter than g_1h_1 and *gh*.

In the last figure on the right, where the pencil ML is held vertical, the plan (if we regard the pencil as a mathematical line) becomes only a point, whether seen on the line AB or represented on a plane looked down upon from above. This point may be marked either *l* or *m* as the vertical projectors through L and M coincide, and the plans of both points are the same.

Exactly the same principles apply to the elevation as to the plan; it is only necessary to repeat the above, using the vertical in place of the horizontal plane. When the pencil is vertical it is parallel to the V.P., and its elevation is equal to it in length; the more the pencil is inclined to the V.P., the shorter its elevation will become, until at last, when the pencil is at right angles to the V.P., its elevation (regarding the pencil as before, as a line only) is a point. The general rule will apply in the case of any plane, whether horizontal, vertical, or inclined in any direction, and may be stated thus:—

"If a line is parallel to a plane, its projection on that plane is equal in length to the line itself; if a line is inclined to a plane, its projection on that plane is shorter than the line itself, in proportion to the angle of inclination; and if a line is perpendicular to a plane, its projection on it is a point."

The converse of the rule is also true. "If the projection of a line upon any plane is equal to the line itself, the line is parallel to the plane; if less, the line is inclined to the plane; and if the projection is a point, the line is perpendicular to the plane."

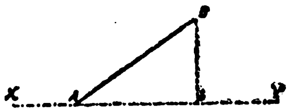
The inclination of a line to a plane is measured by the angle which the line makes with its projection on that plane; thus the angle which a line (or a line produced) makes with its plan (or its plan produced) is the inclination of the line to the horizontal plane, and the angle which a line (or line produced) makes with its elevation (or elevation produced) is the inclination of the line to the vertical plane. If a line is parallel to the H.P.,

its elevation is evidently parallel to XY, because its two extremities are equidistant from the H.P., and therefore the elevations of them are equidistant from XY; similarly, if a line is parallel to the V.P., its plan is parallel to XY.

Problem V.

A line 1″·5 long is inclined to the H.P. at 35°, what is the length of its plan?

Suppose the line to be parallel to a V.P., then its elevation on this plane will be the same length as the line itself, also it will of course make no difference in the length of the plan, whether the line has one end in the H.P. or not.

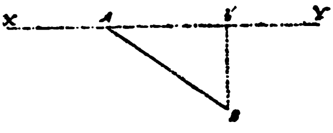


Take a ground-line XY, take in it a point A, and draw AB 3″ long and inclined to it at 35°; through B draw Bb perpendicular to XY; then Ab is the length of the plan required.

Problem VI.

A line 1″·5 long, is inclined to the V.P. at 35°; what is the length of its elevation?

It will evidently make no difference in the elevation whether the line is parallel to the H.P. or inclined to it at any angle; the length of the elevation will depend entirely upon the angle of inclination of the line to the V.P.; nor will it make any difference whether one end of the line is in the V.P. or not. Suppose, therefore, that the line lies



in a H.P. and that one end of it is in the V.P.; take a ground-line XY, take any point in it A, and draw AB 3″ long, and making an angle of 35° with XY, through B draw Bb' perpendicular to XY and meeting it in b'; then Ab' is the length of the elevation required.

The student will see that though Problems V. and VI. are given separately to avoid confusing a beginner, they are really identical, and would both be comprised in the following enunciation :—

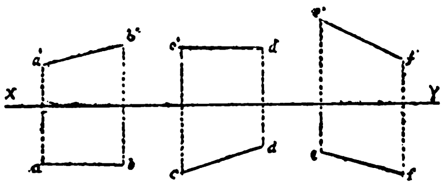
"A line 3″ long is inclined to a plane at 35°; what is the length of its projection upon that plane?"

In the figures below three lines are shown by their projections.

AB is inclined to the horizontal plane because its elevation $a'b'$ is inclined to the line XY, and AB is parallel to the vertical plane because its plan ab is parallel to XY.

CD is parallel to the H.P. and inclined to the V.P.

EF is inclined to both planes.



Problem VII.

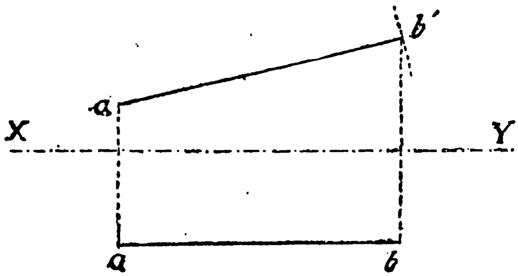
To determine the projections of a line AB, 3″ long, which is parallel to the V.P. and 1″ in front of it; the point A being ·5″ and the point B 1·2″ above the H.P.

As the line is parallel to the V.P., its true length and its inclination will be shown in the elevation: take any point a' at ·5″ above XY to represent the elevation of A, and draw a

parallel to XY at 1·2″ above it, then the elevation of B must be in this line, and we know that the line is three inches long.

From the centre a' with radius 3″ describe a circle cutting this line in b', then b' will be the elevation of B, and $a'b'$ the elevation of the line AB. It may be drawn upon either side of a' and evidently there will only be two lines which will satisfy the given conditions.

Draw through a', a line at right angles to XY, then the plan of A must be in this line; draw a line through b' at right angles to XY; then the plan of B must be in it. Draw a line parallel to XY at 1″ from it, then the plan of AB must be in this line, because the line AB is 1″ from the V.P. and parallel to it, and ab the portion intercepted between the two lines from a' and b' is the plan required.



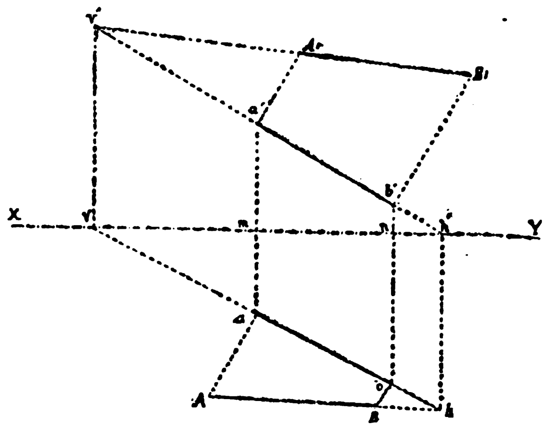
PROBLEM VIII.

The projections of a line AB being given, determine its traces, its actual length, and its inclination to both planes of projection.

If a line is not parallel to a plane, it either meets it or would do so if produced far enough either in one direction or the other. The point where it meets the plane is called, as before explained, the trace of the line on that plane.

Let *ab* and *a'b'* be the projections of the line AB, it is evident that it will have both a vertical and a horizontal trace, because it is not parallel to either plane, neither its plan nor its elevation being parallel to XY.

As the H.T. (horizontal trace) is a point somewhere in the H.P., its elevation must be in XY; and as it is a point in the line AB (or in AB produced) the elevation of the H.T. must be in the elevation of AB (or of AB produced). Therefore, as the elevation *a'b'* does not meet XY, produce it until it meets it in *h'*; then *h'* is the elevation of the H.T. Through this point draw a



line at right angles to the ground-line; then the plan of the H.T. must be in this line, because the plan and elevation of any point lie always in a straight line at right angles to the ground-line. Also, as the H.T. is a point in the line AB (or AB produced), its plan must be a point in the plan of AB (or of AB produced), produce the plan to meet the perpendicular line drawn through *h'* in *h*; then this is the *plan*, that is to say, *h* is the H.T. required, the plan of any point in the H.P. being the point itself.

Similarly, produce the plan ab to meet XY in v; then this is the plan of the V.T., and by drawing through it a line perpendicular to XY to meet $b'a'$ (or $b'a'$ produced), v' the V.T. is found.

Next, to find the true length of AB; imagine this line to be in its true position over its plan ab supported by two vertical projectors, one from A to a and the other from B to b, then we shall have a trapezium AabB. The student should cut a piece of paper in the form required, making one edge of it equal to the plan ab and the two edges or right angles to this equal to $a'm$ and $b'n$, which are the heights of the two ends A and B above the H.P., and stand this piece of paper upon the plan ab; then the upper edge of this trapezium will represent the actual line AB in its proper position. If this trapezium is turned down upon the paper, the edge corresponding to ab being still kept in contact with it, we shall obtain the figure aABb, as shown. This figure is formed by drawing through a and b two lines at right angles to the plan ab and making aA and bB equal respectively to ma' and nb'; then if A and B are joined, AB gives the true length of the original line as required.

In a similar way this might have been found from the elevation by drawing $a'A_1$ and $b'B_1$ equal to ma and nb, which represent the distances of A and A from the *vertical* plane, and if the drawing is carefully constructed the student will find that A_1B_1 gives the same length as already obtained for AB.

Lastly, having produced the line AB to meet its plan produced in h (which it will necessarily do, as the H.T. is the point where a line meets the H.P.), the angle Bhb is the inclination of the line AB to the H.P., and if the other line A_1B_1, which was found from the elevation, is produced to meet $b'a'$ produced in v', the angle at v' is the inclination of the line AB to the V.P.

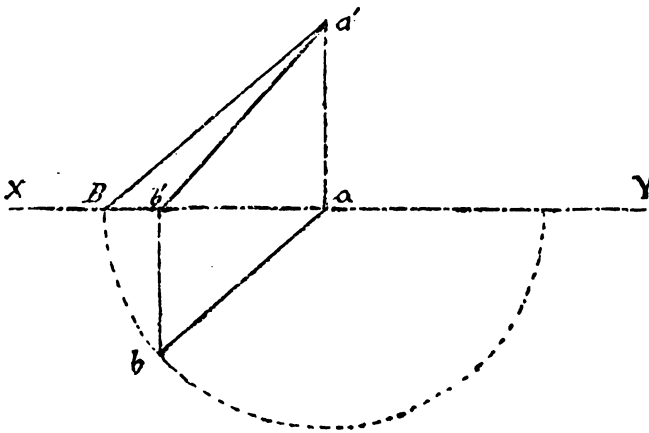
The student will see that the H.T. could have been ob-

tained without producing the elevation by setting up from the plan *a*A and *b*B equal to *ma′* and *nb′*, and producing *ab* and AB till they meet in *h*, and similarly for the V.T.

Problem IX.

A line AB, 3″ long, is inclined at 36° to the H.P., and its plan makes an angle of 20° with XY. Draw its elevation.

In such problems as this it is convenient to consider the line as lying upon the surface of a cone with one extremity of the



line at its apex and the other in the circumference of its base. The cone must be such that its slope, that is, the angle at the base, shall be the given inclination of the line to the H.P.; then any straight line lying upon the surface of the cone from the apex to the base will be inclined at the required angle. The problem is to discover which line will satisfy the other conditions given.

Suppose a cone, the slope of which is 36°, and length of

slope from apex to base 3″, to be placed so that the V.P. passes through its axis, dividing the cone into two equal parts from axis to base. Take a ground-line XY; take any point in it and draw a line B*a*′ 3″ long inclined at 36° to XY; this will represent one side of the cone, *a*′ being its apex. Draw *a*′*a* at right angles to XY, and meeting it in *a*; this will be the plan of the apex and the centre of the base of the cone.

Then if a circle is described with *a* as centre and radius *a*B, this will be the base of the cone, and any radius of this circle will be the plan of a straight line drawn upon the surface of the cone from the apex to the base. Draw the radius *ab*, making an angle of 20° with XY; then this is the plan of a line 3″ long inclined at 36° to the H.P., and the plan is inclined to XY as required. Draw *bb*′ at right angles to XY, and meeting it in *b*′; then as B is a point in the HP., *b*′ is its elevation. Join *b*′*a*′; then this is the elevation of the line of which *ba* is the plan.

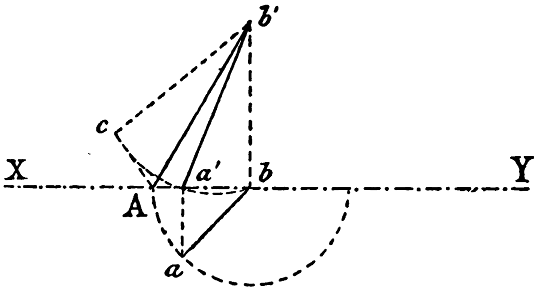
The radius *ab* may of course be drawn to either side, and two radii can be drawn above as well as two below XY, making the required angle with the ground-line. Instead of supposing the V.P. to pass through the centre of the cone, it may be supposed to be at any distance from it by taking the plan of the apex *a* at that distance from XY.

PROBLEM X.

To draw the projections of a line 2″ long, inclined at 60° to the H.P. and at 20° to the V.P.

This problem is very similar to the last. Take a ground-line XY; take any point in it A, and draw A*b*′ 2″ long and inclined at 60° to XY to represent the side of a cone, and draw *b*′*b* at right angles to XY to represent the axis. Draw a line *b*′*c*, making an angle of 20° with A*b*′, and from A draw A*c* at right angles to *b*′*c*; then in this figure A*b*′*c* we have the

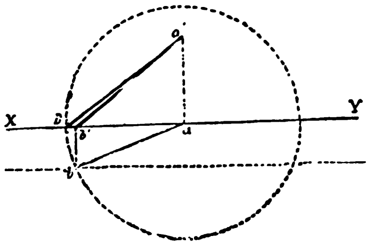
same construction as in Problem VI., and A*b′* being the actual line, *cb′* gives the length of its elevation; but one end of the elevation must be at *b′* and the other in XY, because the line lies upon the surface of the cone. From *b′* as centre describe an arc passing through *c* and cutting XY in *a′* and join *b′a′*; this is the elevation required.



From *b* as centre, with radius *b*A describe a circle which will represent the base of the cone, and from *a′* draw *a′a* at right angles to XY, meeting the circle in *a*. Join *ab*, which will be the plan of the line required.

Problem XI.

A line AB is 1″·5 long; the extremity A is in the V.P.; the extremity B is in the H.P., and 0″·4 in front of XY; the inclination of the line is 40°. Draw its projections.



In XY take a point B, draw B*a′* inclined in 40° to XY and 1·5″ long. From *a′* draw *a′a* at right

angles to XY, and from a as centre, with radius aB describe a circle to represent the base of a cone of which $a'a$ is the axis. Then, as in Problems IX. and X., the line required will be on the surface of the cone, and one radius of this circle will be its plan; but the end B is to be $0''\cdot75$ in front of the V.P. Therefore draw a line $0''\cdot75$ below XY, cutting the circle in b, and join ab; this will be the plan required. Through b draw bb' at right angles to XY, and join $a'b'$; this will be the elevation of the line.

EXAMPLES.

1. Take two points, a and b, 2 inches apart, as the plans of two points of which A is $1\cdot7''$ and B $2\cdot5''$ above the H.P., i.e. above the drawing-paper. What is the true length of the line AB, and what is its inclination to the H.P.?

2. Draw the plan of a line which is $2''$ long and inclined at $40°$; draw its elevation on a vertical plane not parallel to the line.

3. Draw the plan and elevation of a line $2''\cdot5$ long, one end of it being $1''\cdot2$ and the other $1''\cdot8$ above the paper.

4. A line $2''$ long is inclined at $35°$ to the H.P. and at $50°$ to the V.P.; draw its plan and elevation.

5. A line AB $2''\cdot5$ long has one extremity A in the V.P. at $2''\cdot2$ above the horizontal plane; the other extremity B is $1''\cdot5$ above the H.P.; draw its projections when the plan makes an angle of $25°$ with XY.

6. A line $3''$ long is inclined at $20°$ to the H.P.; draw its projections when its plan makes an angle of $45°$ with XY.

7. A line AB is $3''$ long and inclined equally to both planes, its plan is at right angles to XY, the point A is $2''$ above the H.P. and $2''$ in front of the V.P.; draw its projections and traces.

8. Another line CD is the same length and similarly placed to the above, but A is $1''$ in front of the V.P. and $2''\cdot5$ above the H.P.; draw its projections and traces.

CHAPTER IV.

PROJECTIONS OF PLANE FIGURES.

We know by Section III. that if a line is parallel to any plane the projection of the line upon the plane is equal in length to the line itself; and if a plane figure is horizontal, every side of it must be parallel to the horizontal plane, so that the plan of each side must be exactly equal to the side itself. It is evident then that the plan will be equal in length and breadth to the original figure. Similarly, if a plane figure be parallel to the vertical plane, its elevation will be equal to the figure itself.

And if a plane figure be inclined to the horizontal plane, the plans of those lines in it which remain horizontal will still remain of the same length, but those lines which are inclined to the horizontal plane will have their planes shortened more or less according to their inclination. In this case therefore, the plan of the plane figure will preserve its true dimensions in one direction while it will be shortened in the other direction.

The above remark will of course apply to elevations as well as to plans.

And it is evident that if a plane figure is horizontal its elevation will be a line, viz., a portion of the vertical trace of the plane containing the plane figure, and the plan of every plane figure which is vertical will also be a line or portion of the vertical trace of the plane.

PROBLEM XII.

Draw the projections of a square of 1″ side in the following positions:—

i. When it is parallel to the horizontal plane and 0″·3 above it, and two sides are parallel to the vertical plane.

ii. When it is parallel to the horizontal plane and 0″·8 above it, and one diagonal is parallel to the vertical plane.

iii. When two sides are inclined at 45° to the horizontal plane and are parallel to the vertical plane.

iv. When two sides are vertical and the other two inclined at 30° to the vertical plane.

Fig. I.

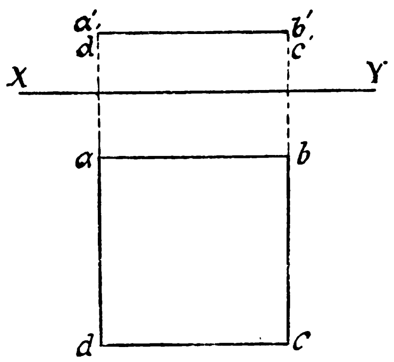
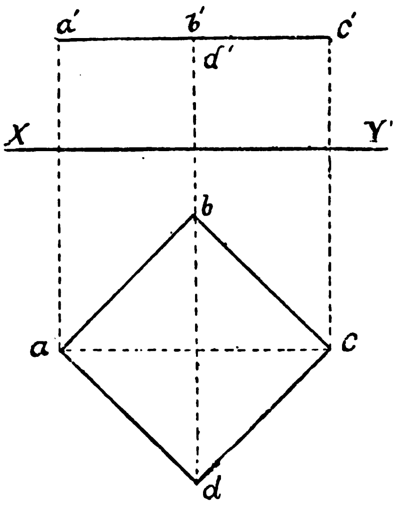


Fig. II.

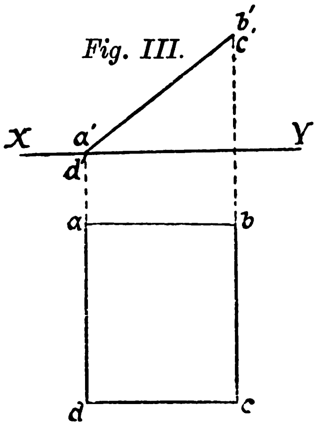


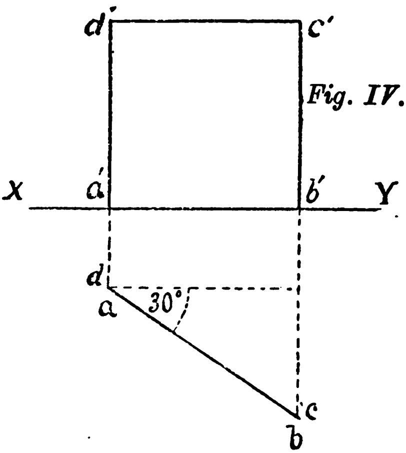
In the first case the plan is evidently equal and similar in every respect to the square itself, and the sides *ab* and *cd* are

parallel to the ground-line, and the elevation is a line equal to one side of the square parallel to the ground-line and $0''{\cdot}3$ above it. The elevation may be marked $a'b'$ or $c'd'$, as one extremity is the elevation of both A and D and the other of both B and C.

As the distance of the square from the V.P. is not given, the distance of ab from XY is not fixed.

In the second the plan is still similar and equal to the square itself, but is placed so that one diagonal ac is parallel





to the ground-line. The elevation is a straight line $a'c'$ parallel to the ground-line and $0''{\cdot}8$ above it, and it is equal in length to the diagonal ac; the centre point of the elevation is evidently the elevation of both B and D.

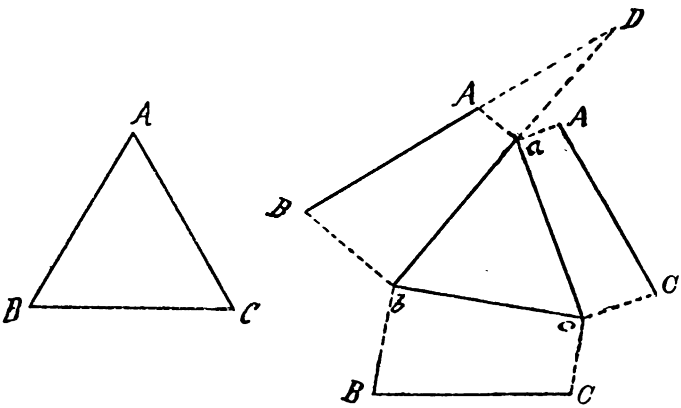
In the third case the elevation must be drawn first; it will be a line $a'b'$ (or $d'c'$) inclined to the ground-line at 45°. Then draw the lines through a' and b' at right angles to XY, and mark off on the former ad equal to $1''$, and draw ab and dc parallel to XY, for it is evident that ad and bc must be equal

to the sides of the square, whilst *ab* and *dc* must be shorter on account of their inclination to the horizontal plane.

The fourth case is the converse of this, the plan *ab* (or *dc*) must be drawn first, and two lines at right angles to XY will give *a'd'* and *b'c'*, which will of course be 1″ long.

Problem XIII.

An equilateral triangle *abc*, each side of which is 2″, is the plan of a certain triangle ABC, and the three points A, B and



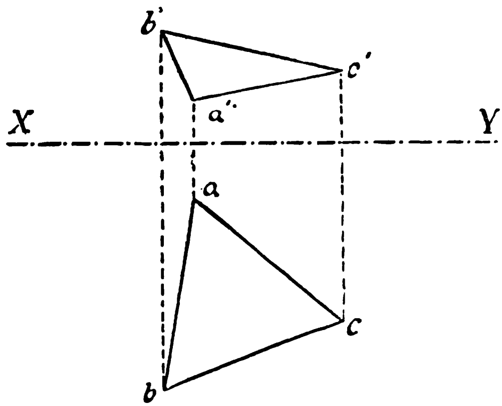
C are respectively 0″·5, 1″·2 and 0″·8 above the H.P.; what is the true shape of the triangle ABC? and what is the inclination of the side AB?

In this problem, as the heights of the three points are given, it is unnecessary to draw the elevation; taking as is usual the surface of the drawing-paper to represent the H.P., on it draw the equilateral triangle *abc*, from *a* and *b* draw lines at right angles to *ab*, and make *a*A equal to 0″·5, and *b*B equal to 1″·2; join AB; then, as shown in the last problem, AB is the actual

length of the side of which *ab* is the plan. Similarly, draw two lines at right angles to *ac* of lengths 0″·5 and 0″·8, and two of 1″·2 and 0″·8 at right angles to *bc*; joining the ends of these, we shall obtain the actual lengths of AC and BC. With these three lengths the triangle ABC can be constructed. Produce *ba* and BA to meet in D, then the angle at D is the inclination of the side BA to the H.P.; if more convenient, this can be found by drawing through A a line parallel to *ab*, the angle at A between this line and AB is evidently the same as the angle at D.

Problem XIV.

Draw an elevation of the triangle, the plan of which is given in the preceding problem, on a V.P. which is not parallel to any one of the sides.

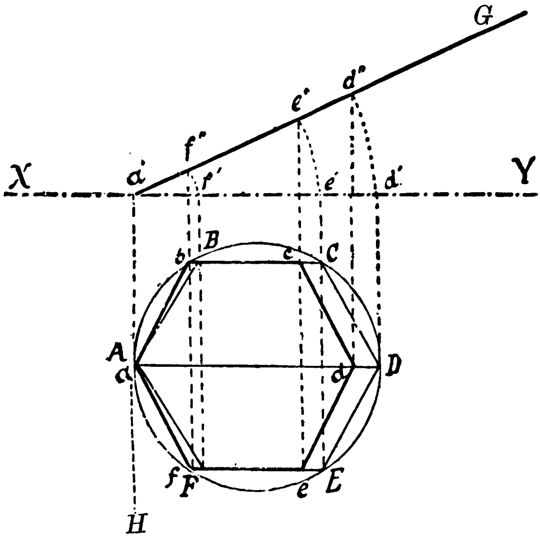


Make the plan *abc* as before. If a V.P. were taken parallel to any one of the sides, the ground-line XY would of course be parallel to that side, as explained before, therefore draw XY so as not to be parallel to any of the sides; then through the

points *a*, *b*, and *c* draw lines at right angles to XY, and on these set off the lengths 0″·5, 1″·2, and 0″·8 respectively from the line XY; these will give *a*′, *b*′, and *c*′ the elevations of the points A, B, and C, and by joining *a*′*b*′, *a*′*c*′, and *b*′*c*′ we get the elevation required.

Problem XV.

Draw the plan and elevation of a hexagon of 1″·25 side, when one diagonal is inclined at 25° to the H.P.



First draw the hexagon itself ABCDEF with one diagonal AD parallel to XY, which would be the plan if the hexagon were lying upon or parallel to the H.P.; then draw the lines A*a*′, FB*f*′, EC*e*′, D*d*′ at right angles to XY. The line *a*′*f*′*e*′*d*′ is the elevation when the hexagon is lying on the H.P.

Draw the line a'G making an angle of 25° with XY, and from centre a' with radii $a'f'$, $a'e'$, $a'd'$ describe arcs cutting this line in f'', e'', and d'', then $a'f''e''d''$ is the elevation of the hexagon when it has been revolved upon the line Haa', until the diagonal AD makes an angle of 25° with the horizontal plane. Draw through f'', e'', and d'' lines perpendicular to XY, then the plans of F, E, and D must be in these lines, and it is evident that the plans of B and C must be in the same lines with the plans of F and E.

Also, as the width of the hexagon will not be changed, the plans of B and C will be in CB or CB produced, and the plans of E and F in EF or EF produced, and the plan of D will also be in DA; therefore *abcdef* will be the plan of the hexagon required.

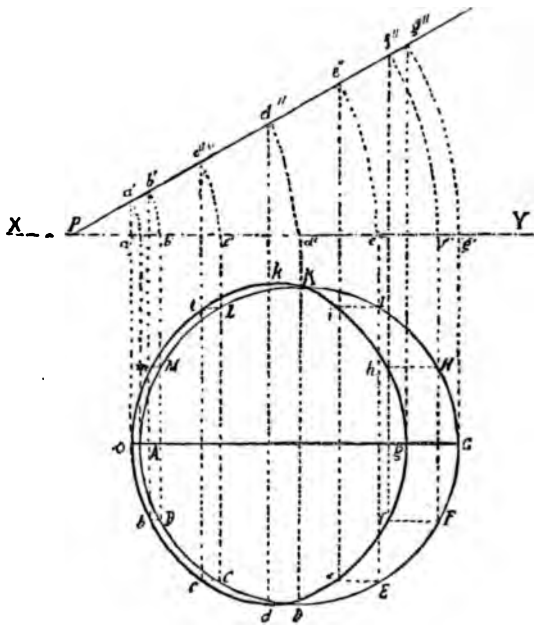
Problem XVI.

A circle is in a plane inclined at 30° to the H.P.; draw its plan and elevation.

In drawing the projections of a curve it is necessary to fix the positions of the projections of several points on the curve, and then to join these points. The closer together the points are taken, the more accurately can the curve be drawn. In this example we will take twelve points, but it will be well for the student to make the diameter of the circle about 4″, and to use sixteen or twenty-four points. Draw a circle ABCDEFGHIKLM, then as the circle is to be part of a plane inclined at 30° to the H.P., it is evident that one diameter of the circle will be inclined at this angle to the H.P.; let AG, which is parallel to XY, be the diameter to be so inclined. The circle, considered as a twelve-sided figure ABCDEFGHIKLM, is then to be treated exactly as the hexagon in the preceding problem; first draw through all the points lines perpendicular to XY, which will give the elevation of the circle when it is lying in the H.P., then by means of arcs transfer these elevations of ABC,

&c., to a line drawn at 30° with XY; we thus get a'', b'', c'', d'', e'', f'', g'' (b'', c'', d'', e'', f'' being the same in this case as m'', l'', k'', i'', h''), the elevation of the twelve-sided figure when inclined at the given angle.

From this elevation we get the plan required exactly as in



the preceding problem, vertical lines are drawn through a'', b'', c'', d'', &c., horizontal lines through A, B, C, D, &c., and the intersections of these lines give the points *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *k*, *l*, *m*, through which a curve (which will be an ellipse) is drawn.

In such problems as this and the preceding it is immaterial whether the line in the elevation, which is drawn at the required angle with XY, meets XY in a' as drawn in Problem XV. or in any other point, as P in the figure above.

In the above problems there has been no difficulty in arranging the figure so that the plane containing it should be perpendicular to the V.P., both when horizontal and when inclined to the H.P. at any angle. When this cannot be done, it is best to use a second V.P., generally at right angles to the first, so that planes which are perpendicular to the one V.P. must be parallel to the other. This will be easily understood from the following problem.

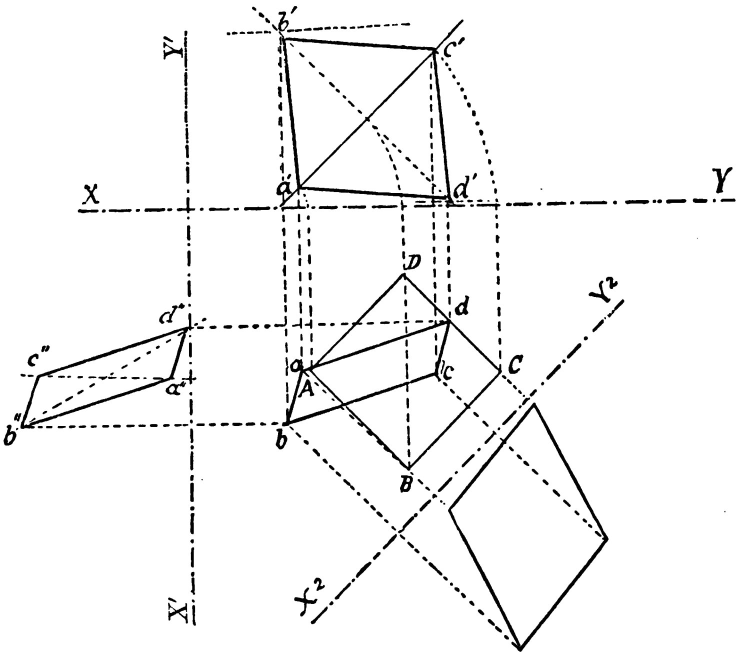
Problem XVII.

A square has one diagonal inclined to the H.P. at 45°, and the other at 60°; draw its projections.

Draw the square ABCD with the diagonal AC parallel to XY, and let this be the diagonal to be inclined at 45° to the H.P. Draw lines perpendicular to XY, and by means of arcs transfer the elevations of a and c to a line making an angle of 45° with XY; then if the other diagonal bd were horizontal, that is if the plane of the square were perpendicular to the V.P., the elevations of B and D would both be at the point midway between a' and c'; but BD is to be inclined at 60°, so that one end (say B) is to be above and the other below the line $a'c'$, and as the shape of the square itself does not vary, it is evident that they will both lie in a line drawn through the centre of $a'c'$ at right angles to it. We have only therefore to discover what will be the height of B above and of D below the line $a'c'$.

Take a second ground-line X'Y' at right angles to XY, and

therefore parallel to the diagonal BD. With this ground-line make an elevation of the diagonal AC, making the heights of a'' and c'' above X'Y' equal to the heights of a' and c' above XY. Through the centre of $a'c'$ draw a line inclined at 60° to X'Y', then as the diagonal BD is parallel to this V.P. the



elevation $b''d''$ will be equal to BD, and the two parts of it on either side of $a''c''$ will evidently be equal, therefore take b'' and c'' at the same distance from the point of intersection with $a''c''$ as B and D are from A.C. Join a'', b'' c'', d''. This is one elevation of the square when inclined as required; from this elevation the other can if necessary be completed. Take the heights

of b'' and d'' above $X'Y'$, and mark b' and d' at similar heights above XY. Join a', b', c', d'; this is the elevation upon a V.P. parallel to AC.

The plan is constructed by drawing lines perpendicular to the ground-lines from a', b', c', d', and a'', b'', c'', d''; the intersections of the lines from a' and a'' will give the plan a, and similarly for the other three.

Any number more elevations can be drawn on different vertical planes in any positions; for instance, if an elevation is required upon a V.P. parallel to the line BC, draw a ground-line X^2Y^2 parallel to BC, draw lines at right angles to X^2Y^2 through the several points of the plan, and on these mark off the heights already obtained for them in the first and second elevations.

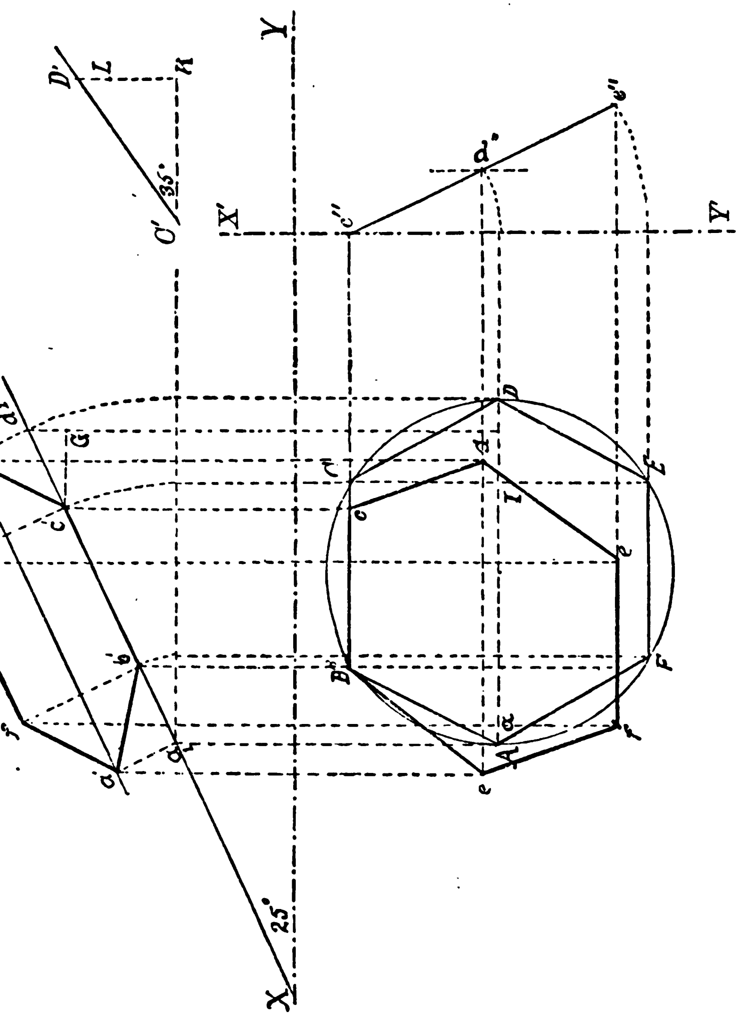
Problem XVIII.

A regular hexagon has one side inclined at 25°, and the adjacent side at 35° to the H.P.; draw its projections.

Draw a hexagon ABCDEF; suppose BC to be the side which is to be inclined at 25° and CD at 35°, take XY parallel to BC and draw a line inclined to it at 25°; by means of lines at right angles to XY, and arcs as in the preceding problems, find upon this line the points a_1, b', c', d_1, which would be the elevation if the plane of the hexagon were inclined to the H.P. at 25°, that is to say if the lines BF and CE were horizontal.

Through c' draw a horizontal line c'G, meeting in G a line drawn from d_1 at right angles to XY, then d_1G would be the height of the point D above C, or, what is the same thing, above the point I where CE meets AD, or in fact above any point in CE if this were a horizontal line.

By the conditions of the problem, however, the lines BF and CE are inclined upwards from B and C, the hexagon


X
Y
X'
Y'
25°
35°
C'
D'
L
K
c''
d''
e''
D
E
F
A
G
I

being inclined so that CD makes an angle of 35° with the H.P.

What has to be found therefore is the height of D, under these circumstances, above C, which when known will give the inclination of CE or BF. Take a line C′K (see the small figure); draw C′D′ making an angle with it of 35° and equal to CD; from D′ draw D′K at right angles to C′K; then D′K is the height of D above C, when CD is inclined at 35° to the H.P. Mark off on it D′L equal to d_1G, then the remainder LK is the height of I above C when the hexagon is so inclined.

Take another ground-line X′Y′ at right angles to XY, draw a line parallel to it, and at a height above it equal to LK. Draw a line through C at right angles to X′Y′ and meeting it in c''; draw another from D, and through the point where it meets X′Y′ with centre c'' describe an arc cutting the line parallel to X′Y′ in d''. Draw a line through c'' and d''; this line will show the inclination of the line CE when the given conditions of the problem are fulfilled. By means of another line from E at right angles to X′Y′ and another arc find e'', or simply take $c''e''$ equal to CE. The height of e'' above X′Y′ is evidently twice that of d'', as EI is equal to IC, and these are the heights of the diagonal AD above BC, and of the side FE above AD; using the heights thus obtained, draw two parallel lines above the inclined line in the first elevation, and by lines drawn through a_1, b', c', d_1, at right angles to these lines fix the points a', f', e', d', and complete this elevation. Then lines drawn through the points of this elevation at right angles to XY, by their intersections with lines from c'', d'' and e'' at right angles to X′Y′, will give the plan of the hexagon.

Examples.

1. Draw the plan and elevation of a pentagon of $1''$ side en one side is horizontal and the pentagon is in a plane lined at 35° to the horizontal plane.

2. An octagon has one diagonal inclined at 50° to the izontal plane; draw its plan and elevation.

3. An equilateral triangle is inclined to the horizontal at h an angle that the plan of its perpendicular is one-third he original length; draw its plan and elevation.

4. An ellipse which has a minor axis of $1''\cdot33$ is the plan circle of $1''$ radius; draw the plan and elevation. What he inclination of the plane of the circle?

5. A rectangle is $2''\cdot2$ long, and $1''\cdot1$ wide; draw its projec-ns when its plan is a square.

6. *abc* is the plan of an equilateral triangle; $ab = 2''$, *bc* d *ac* each $= 1''$. Both A and B are $0''\cdot8$ above the H.P.; at height is C?

7. A square ABCD of $2''$ side has one corner A in the P., B is $0''\cdot25$ and C $1''$ above it; what height is D? raw the plan, and an elevation on a V.P. parallel to the le of the plan *bc*.

8. An octagon has one axis inclined at 30°, and a side djacent to this axis is inclined at 50° to the H.P.; draw its projections.

9. ABC is an isosceles triangle, $AB = AC = 2''\cdot5$, $BC = 1''\cdot25$; AD is a line from the apex to the centre of the base; BC s inclined to the H.P. at 35°, and A is $1''$ higher than D; draw the plan and elevation.

10. An octagon has one side horizontal, and the sides cent to it inclined to the H.P. at 15°; draw its pro-ons.

11. A hexagon has one side horizontal and one diagonal inclined to the H.P. at 20°; draw its projections.

12. A parallelogram has sides of 1″·2 and 2″·5, the shorter sides are inclined at 20° and the longer at 30°; draw its projections.

CHAPTER V.

ON PROJECTIONS OF SOLIDS.

THE solids commonly used to illustrate the principles of solid geometry are the cube, prism, pyramid, sphere, cylinder, cone, tetrahedron, and octahedron.

A cube has six equal faces, each of which is a square.

A prism has two bases, that is to say, a base and a top both equal and similar, such as two triangles, two squares, two pentagons, &c., each side of the top being connected with one side of the bottom by a four-sided figure called a face of the prism.

If the faces are perpendicular to the top and to the base, the prism is called a right prism; if not, it is an oblique prism. The line joining the centres of the base and the top is called the axis.

A pyramid has one figure (such as a triangle, square, pentagon, &c.) for a base, and to each side of this base is joined a triangular face, all the faces meeting in a point called the apex. The line joining the apex to the centre of the base is called the axis. If the axis is perpendicular to the base it is a right pyramid; otherwise it is oblique.

Prisms and pyramids are named according to the shapes of their bases; thus a "square prism," a "triangular pyramid," a "pentagonal prism," a "hexagonal pyramid," &c.

A sphere or globe is a solid the surface of which is at every point equidistant from the centre. A sphere may be supposed to be generated by the revolution of a circle or semicircle upon its diameter.

A cylinder may be considered to be a prism with an infinite number of faces. Its two bases (i.e. its base and top) are circles (i.e. they are polygons with an infinite number of sides). We may suppose a cylinder to be generated by the revolution of a rectangle upon one of its sides.

A cone may be defined as a pyramid with an infinite number of faces; the base is a circle. We may suppose a pyramid to be generated by the revolution of a right-angled triangle about its perpendicular.

A tetrahedron is a solid having four equal faces, each of which is an equilateral triangle. It is in fact a triangular pyramid of which each side is equal to the base.

An octahedron has eight equal faces, each of which is an equilateral triangle, i.e. it consists of two square pyramids placed base to base, the faces of each pyramid being equilateral triangles.

In order to draw the projections of a solid it is only necessary to draw the projections of each of its faces according to the rules already given in Section IV.; and in regular solids, as those above described, the process may be much shortened.

PROBLEM XIX.

Draw the projections of a cube of 1″ side, ABCDEFGH, standing on the horizontal plane with one side parallel to the vertical plane. And again, when one face is inclined at 30° to the V.P.

Draw a square *abcd* of 1″ side, with one side *ab* parallel to the ground-line XY; this will be the plan of the cube, and if we draw perpendiculars from *a* and *b* to XY, i.e. if we produce *cb* and *da* to meet the ground-line in c' and d', the portion of the ground-line $d'c'$ will be the elevation of the base of the cube. And if we draw two lines 1″ long, $d'h'$ and $c'g'$, perpendicular to XY, these will be the elevations of the two sides of the cube BFGC and AEHD, and the line from h' to g' will be

Fig. I.

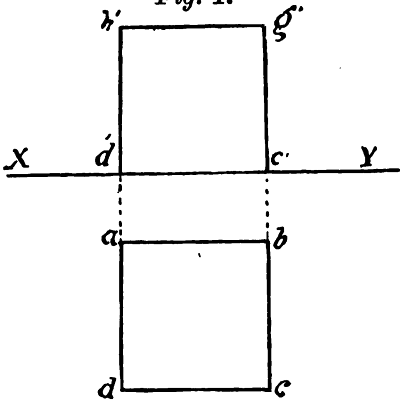
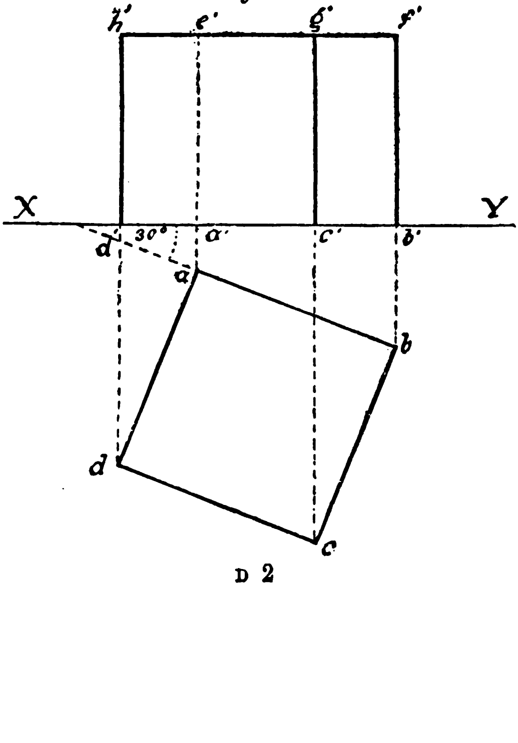


Fig. II.



the elevation of the top EFGH, and $c'g'h'd'$ is the elevation of the face CGHD or of the face AEFB, for it will be seen that the elevation of CGHD will cover the elevation of AEFB.

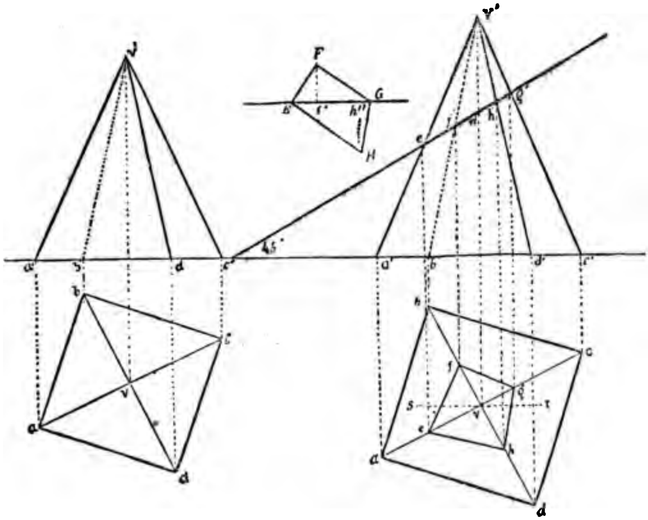
Again, suppose the same cube with its base in the horizontal plane and one face inclined to the vertical plane at 30°; we must first draw the plan *abcd* with one side as *ab* inclined at 30° to the ground-line; then perpendiculars from the angles to XY will give us the elevations of these points $a'b'c'd'$, and at one inch above these will be the elevations of the angles of the top side $e'f'g'h'$; then $c'g'h'd'$ will be the elevation of the side CGHD, and $c'g'f'b'$ is the elevation of CGFB, and the elevation of the other two faces AEHD and AEFB will be behind these, and therefore will not be seen; for this reason the edge $a'e'$ is shown by a dotted line

Problem XX.

A square pyramid stands upon the H.P., and one edge of the base is inclined at 20° to the V.P. The base is 1″·3 side and the height of the pyramid is 2″; draw its projections. Also draw the plan of a section of the pyramid made by a plane inclined to the horizontal plane at 45° and passing through the axis at 1″·3 above the base, and show the true form of this section.

Draw a square *abcd*, with one side *bc* inclined at 20° to XY; draw the diagonals *ac*, *bd*; then the point *v* where they intersect is the plan of the vertex of the pyramid; and if vv' be drawn perpendicular to XY and v' be placed 2″ above XY—v' will be the elevation of the vertex. The elevations corresponding to the plans *a*, *b*, *c*, *d*, will be found as in Problem I., and the lines $v'a'$—$v'd'$—$v'c'$ will be the elevations of the edges of the pyramid VA, VD, and VC. The edge VB is behind the pyramid, and therefore its position is shown by $v'b'$, a dotted line only. And it is evident that *va*, *vb*, *vc*, *vd* are the plans of the edges VA, VB, VC, VD.

Next, if we take a point in the elevation of the axis of the pyramid 1″·3 above XY, and draw through it a line making an angle of 45° with XY, this will represent a plane seen edgewise which is inclined at this angle to the H.P.; and the points e', f', g', and h', in which this line meets the lines $a'v'$, $b'v'$, $c'v'$, and $d'v'$, are the elevations of the points in which the inclined plane meets the edges of the pyramid AV, BV, CV, DV.



To find the plan of one of these, from e' draw a line perpendicular to XY; then the plan of E must also lie in this line. It must also lie in *av*, which is the plan of the edge AV, for the point E is a point in this edge; therefore the intersection with the plan *av* of the line drawn from the elevation e' is the plan *e* which is required. In the same way the plans *f*, *g* and *h* are found, and by joining these we get *efgh*, the plan of the section EFGH.

To find the true shape of this section it must be observed that the dimensions of it in one direction, parallel to XY, are shown in the elevation, as in the case of the square shown in Fig. III., Problem XII., Chapter IV., or in the hexagon in Problem XV., where the lengths in the elevation $a'f''e''d''$ (which are the same as in the elevation $a'f'e'd'$) show the actual dimensions of the hexagon in the direction parallel to XY. The widths or dimensions in the direction at right angles to XY are shown in the plan of the section *efgh*, as the fact of the section being on a plane inclined at 45° does not affect the dimensions in this direction, as will also be seen by referring to the figure of the hexagon in Problem XV.

To avoid confusing the second figure, draw again, separately the elevation of the section, making EG equal to $e'g'$, and taking f'' and g'' at the same intervals as f' and h', and through these points draw lines at right angles to it; also through *v*, the plan of the vertex, draw a line *st* parallel to XY; from f'' take f''F, and from h'' take h''H, equal to the distances in the plan of *f* and *h* respectively from the line *st*. Join EFGH; this gives the true form of the section.

Problem XXI.

Draw the projections of a cone, diameter of base 3″·5, height 5″; make a section of it by a plane inclined to the H.P. at 50°, and dividing the axis into two equal parts; show the plan of this section and also its true form.

The plan of a cone is of course a circle, the centre of which shows the apex, and the elevation is a triangle. Draw accordingly a circle of 3″·5 diameter; take any ground-line, and upon it place a triangle with the apex in line with the centre of the circle and the base equal to the diameter, and make the height of the triangle 5″: these will be the plan and elevation of the cone.

Through the centre of the axis draw a line inclined at 50°

to XY, cutting the elevation of the cone in r' and z'; it is required to draw the plan of the section made by this plane. To do this it is necessary to imagine the cone to be a pyramid with a many-sided base, and to take in the circle a number of points as the angles of this base, as was the case in dealing with the circle in Problem XVI.

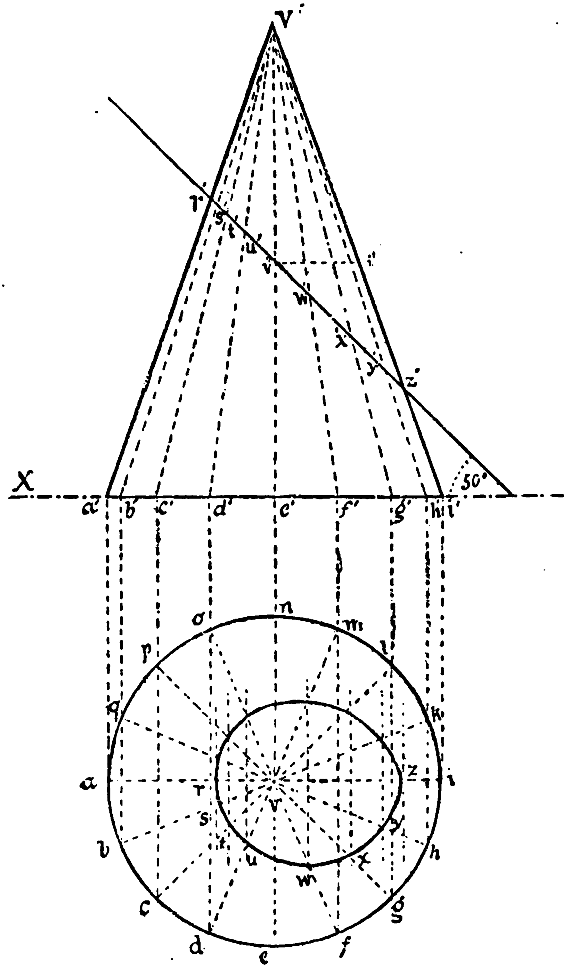
Take any number of points in the plan of the base, as a, b, c, d, &c., and draw through them diameters of the circle, which will be the plans of the edges of the pyramid. Find the elevations a', b', c', d', &c., of these points of the base, and draw from them lines to the apex, which will be the elevations of the edges.

Let the line inclined to the H.P. at 50°, passing through the centre of the axis v'', meet the elevations of the edges of the pyramid in $r', s', t', u', w', x', y', z'$. From these draw lines at right angles to XY to meet the plans of the same edges in r, s, t, u, &c. A curve drawn through these points will give the plan of the section, and the greater the number of sides which have been assumed for the pyramid, i.e. the more numerous the points a, b, c, d, &c., the more accurately will this curve be defined.

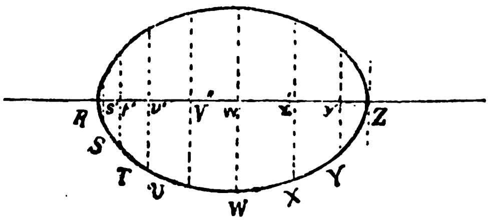
The true form of the section being required, it will be seen that the dimensions in one direction, i.e. the lengths, are given by the points in the elevation of it, r', s', t', u', &c., while the dimensions in the other direction, i.e. the widths, are given by measuring the widths at the corresponding points in the plan.

Therefore, through s', t', u', &c., in the elevation (or in a separate figure to avoid confusing the drawing), draw lines at right angles to the inclined line, and on these mark off the corresponding widths from the plan; that is to say, take S at the same distance from the inclined line that s in the plan is from the diameter ai, and so on for the other points. Then the curve R, S, T, U, &c., is the actual form of the section.

It will have been seen in drawing the plan that no width


v'
r'
s'
t'
u'
v'
w'
x'
y'
z'
X
50°
a'
b'
c'
d'
e'
f'
g'
h'
i'
n
o
m
p
l
q
k
a
r
z
i
v
s
y
b
t
u
x
h
w
c
g
d
e
f

is fixed for the curve at the line joining v and v' as the elevation of the edge $v'e'$ is in one line with the plan of the same



edge, and therefore a line drawn from v'' does not cut the plan as is the case with the others.

It is easy, however, to fix the width at this point, as the point v'', being in the axis of the cone, will be equally distant from any point on its surface at the same height above the H.P.; therefore, through v'' draw a line parallel to XY to meet the elevation of one side of the cone $v'i'$; the length of this line will be the half-width to set off in the plan from v on ve and on vn, and it will be the width to set off on both sides of v'' in drawing the true shape of the section.

Problem XXII.

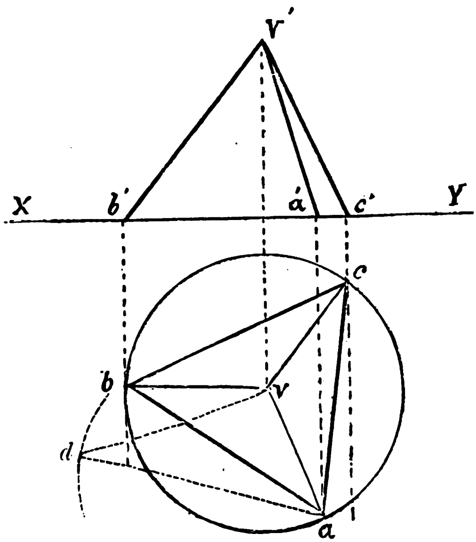
Draw the projections of a tetrahedron of 1″ edge with one face horizontal.

Draw an equilateral triangle of 1″ side abc—this will be the plan of the horizontal face of the tetrahedron; and draw lines from the three angles to meet in the centre v—these lines will be the plans of the three sloping edges of the tetrahedron, and v will be the plan of the vertex. In the elevation $b'a'c'$ is the elevation of the horizontal face, and the elevation of the vertex must be in the line drawn through v at right angles to XY. It is necessary to find the height of the vertex above the horizontal plane.

It will be seen that the vertex is at the apex of three right-

angled triangles, each of which has the plan of one of the sloping edges *av*, *bv* or *cv*, for its base, and the edge itself *a*V, *b*V, or *c*V for its hypotenuse.

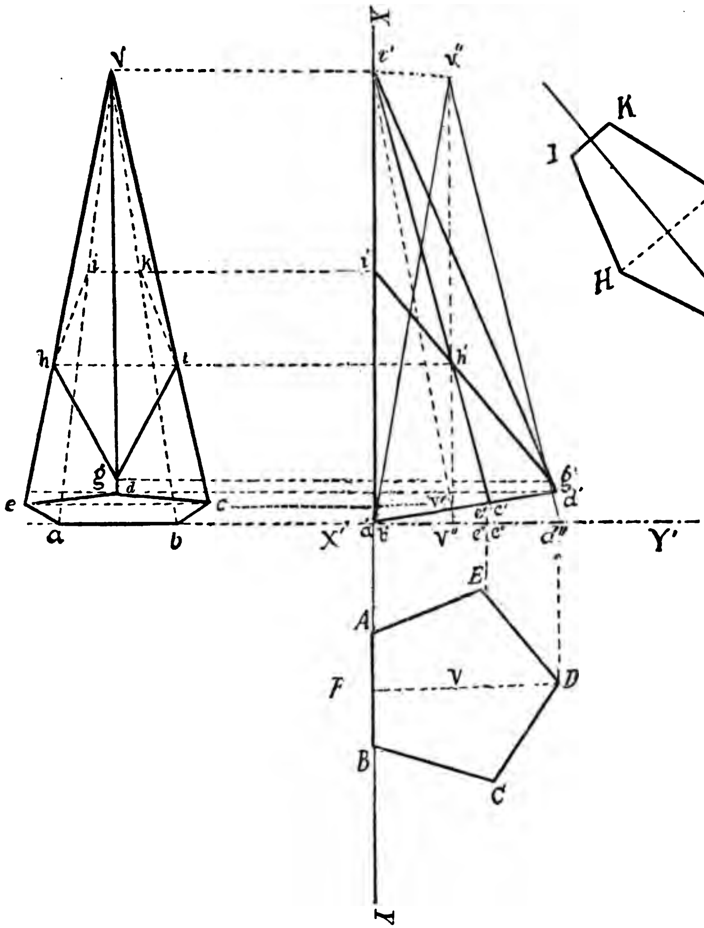
From *v* draw a line perpendicular to *av*, and from *a* with radius of 1″ draw a circle cutting this line in *d*; this triangle *avd* will represent one of the above-mentioned triangles when it is revolved upon its base *av* until it becomes horizontal, and



therefore *vd* will represent the height of the vertex V above the horizontal plane; make the height of *v*′ in the elevation above XY equal to *vd*, then *v*′ will be the elevation of the apex. Join *v*′*a*′, *v*′*b*′, *v*′*c* to represent the elevations of the edges.

Problem XXIII.

A pentagonal pyramid of 1″·25 side of base and 4″·75 height lies upon one of its faces; draw the plan and elevation



v
i
k
h
l
g
d
e
c
a
b
X
v'
v''
i'
h'
g'
d'
v'
e' c'
X'
a' b'
v''
e' c'
d''
Y'
K
I
L
H
G
E
A
D
F
v
B
C
Y

when the axis of the pyramid is parallel to the V.P., also show a section made by a plane inclined at 60° to the base of the pyramid, and draw the true shape of this section.

In this problem, as in No. XVII., a *subsidiary* ground-line is necessary, that is to say, in order to obtain certain dimensions required we must make a subsidiary elevation on a V.P. which is in a different position to the one on which the required elevation will be drawn. The simplest projection of a pyramid which can be drawn is the plan of it when standing on its base, and this is the same thing as an elevation of it on a V.P. at right angles to its axis when this is horizontal.

Draw a pentagon of $1''\cdot25$ side, and draw a line DF from one of the angles to the centre of the opposite side (which will also pass through the centre point of the pentagon); take a ground-line X_1Y_1 parallel to DF, and draw lines at right angles to it from the angular points of the pentagon A, B, C, D, E, and from the central point V, and complete the elevation of the pyramid $a'v''d''$, making the height $v''V''$ equal to $4''\cdot75$. This is the elevation on a plane parallel to the axis of the pyramid when the axis is horizontal; but by the conditions of the problem the pyramid is to lie upon one face, take therefore another ground-line XY in the same line with AB, and therefore containing the elevations a' and b', and incline the elevation of the base $a'd''$ until the face $a'v'$ comes into XY. This is done by taking on XY $a'v'$ equal to $a'v''$, striking arcs from centre v' with radius equal to $v''d''$, and from centre a' with radius $a'd''$, the intersection of these arcs will give d', and then arcs struck from the same centre a' through the points c'' (which is the same as e'') and V'' will give on the new elevation $a'd'$ the points c' (or e') and V'. Join $c'v$ to represent the edge, and the line $V'v'$ may be dotted to represent the axis. This gives the elevation of the pyramid in the required position, and to make the plan the lengths must

be taken from this elevation, and the widths from the pentagon A, B, C, D, E.

From the points v', a', c', and d' draw lines at right angles to XY, take a line parallel to it for the axis or central line of the plan, and set off the widths, marking b and a, c and e at distances from the central line equal to those of B and A, C and E from the line DF; join ab, bc, cd, de, ea to complete the base, and vc, vd, ve for the edges; the lines va and vb should only be dotted as these two edges are underneath the solid.

For the required section take a point in the line $v'V'$ and through this draw a line inclined at 60° to $a'd'$ and meeting the edges $d'v'$, $c'v'$, $a'v'$ in g', h', and i'. This is the elevation of the section. From these three points draw lines at right angles to XY, to meet the corresponding lines in the plan in g, h (or l), and i (or k); join gh, gl, and by dotted lines hi, ik, and kl, this gives the plan of the section. To find its true shape take the line $g'i'$ in the elevation (or to save confusing the figure take another line equal to it), and through h' and i' draw lines at right angles to it, and on these mark off the widths of the section, making HL equal to hl in the plan where the true width at this part is shown, and IK equal to ik; GHIKL is the form of section made by the given plane.

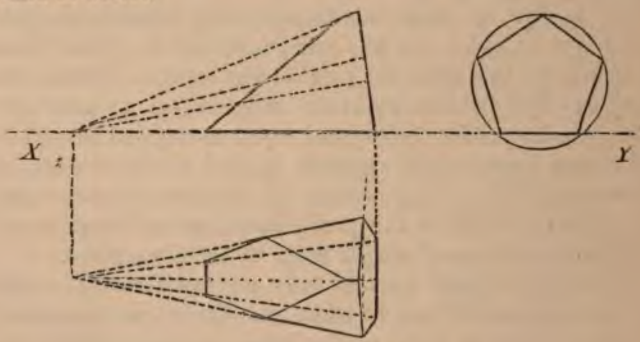
(Note. The student must carefully observe that the widths for this section could not be taken from the original pentagon, as might be done in the case of a prism, as the width of the pyramid diminishes from the base to the apex, so that the width between the two edges at h' is less than at c', and similarly at i' where ik is still further reduced.)

Problem XXIV.

A pentagonal pyramid of 1″·25 side of base and 4″·75 high lies upon one face, with its axis parallel to the V.P., and has the upper portion cut off by a plane which passes through the

axis at a point 2″ above the base, and is inclined to the base at 60°. Draw the projections of the truncated pyramid.

The conditions of this problem are the same as in the last, except that only the truncated pyramid (that is to say, the lower portion after the top is cut off) is to be shown. The dimensions given are the same in order to save repetition, and the problem is to be worked in precisely the same way, except that the upper portion of the pyramid in both the elevation and plan, which must be drawn in order to get the section, is to be omitted when the drawing is inked in or shown only by dotted lines, the plan of the section *ghikl* being drawn with firm lines. The actual shape of the section may of course be found as before.



PROBLEM XXV.

The annexed Figs. I. and II. are elevations of a prism, one being on a V.P. parallel to a long edge, and the other on a V.P. parallel to the base of the prism. The line TT′ represents a plane cutting it. Draw a plan of the remaining portion of the prism when the part to the left of TT′ is removed, and show the true form of the section.

If the elevations were not lettered (as they would probably not be in an examination paper), it would be easy by drawing

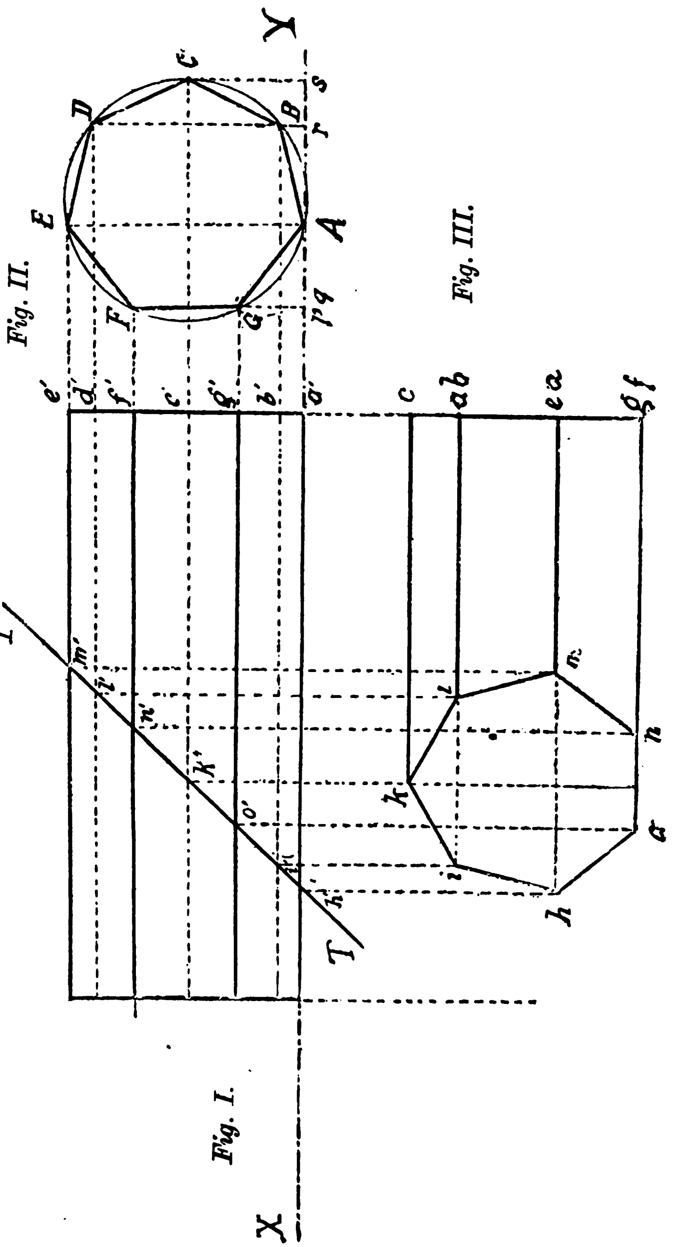


Fig. I.

Fig. II.

Fig. III.

lines parallel to XY to discover that the points marked A, B, C, D, E, F, G in the one correspond to those marked a', b', c', d', e', f', g' in the other, and the corresponding edges are cut by the intersecting plane in the points h', i', k', l', m', n', o' respectively. When the portion of the prism to the left of TT' is removed, the irregular heptagon $h'i'k'l'm'n'o'$ becomes the end of the prism; it is required to find the plan of it.

From these points draw lines at right angles to XY—the plans of the points must of course be in these lines; also continue the elevation of the other end of the prism $e'a'$, which is a line at right angles to XY—the plan of the same end will lie in this line. Thus the positions of all the points required are known so far as regards measurements made parallel to XY, those in the opposite direction must be found from the other elevation. From the points A, B, C, D, E, F, G draw lines at right angles to XY, and meeting it in p, A, r, s (if the heptagon were in a different position these points might be seven, in this case they are only four in number); on the line which contains the plan of the end of the pyramid mark off the points f or g, a or e, b or d, and c at intervals equal to those between p, A, r and s, and through these draw lines parallel to XY to represent the edges of the prism. The intersections of these lines with those drawn from h', i', k', l', m', n', and o' will give the points h, i, k, l, m, n, o, by joining which the plan of the other end of the prism is constructed.

In both the elevation and the plan, lines representing edges which are hidden by the prism itself are shown by dotted lines.

To find the true form of the section take the line TT', and through the points h', i', k', l', m', n', o' draw lines at right angles to it, and set off upon these the distances of the corresponding points in the plan from the central line of the prism; or the distances may be taken from the line XY, as the *difference* between the distances of h and i or h and k for example from

XY will be the same as the *difference* between their distances from the central line of the prism, or of *any other line parallel to it.*

Problem XXVI.

The figure on next page represents a truncated cone. Draw a plan of it, and show the true form of the section where the cone is truncated.

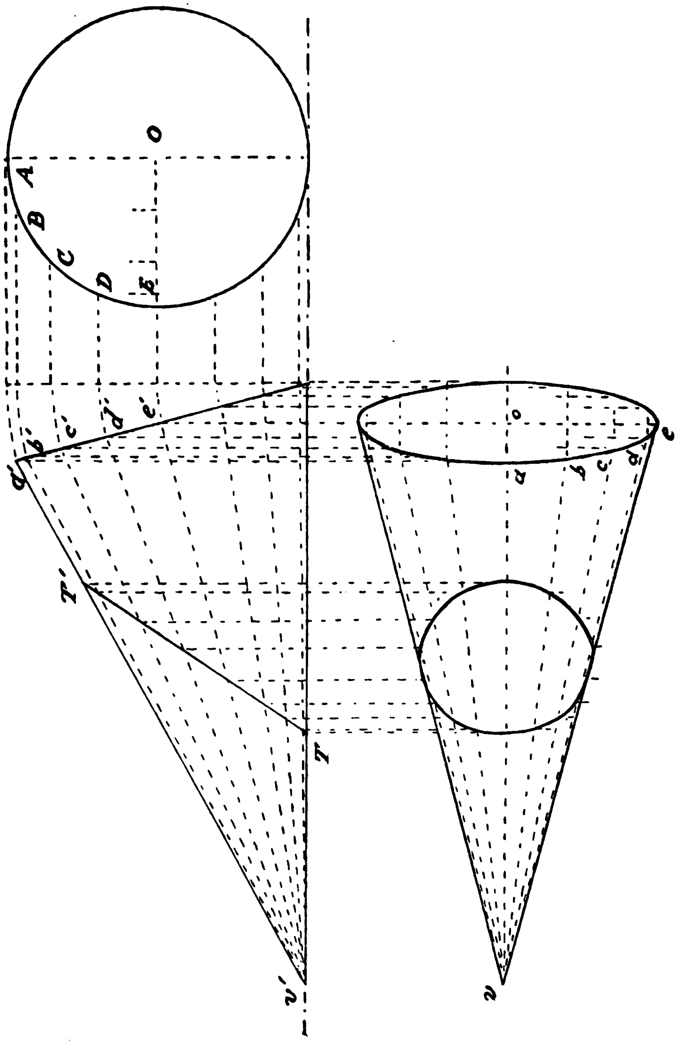
This problem is again similar to No. XXI; a polygonal pyramid must be supposed to occupy the place of the cone, and the more faces this pyramid is supposed to have, that is to say the closer together the points are taken in the base, the more accurately will the curve be found in the plan. The points in the elevation a', b', c', d', e', f', &c., are found first as before, lines perpendicular to XY are drawn from these, and then an axis ov being drawn parallel to XY, distances are set off on these lines from the axis equal to the distances of the points A, B, C, D, E, F from the line AO, and thus the points a, b, c, d, e, f are found.

To find the section made by the line TT′ the construction is precisely similar to that in Problem XXI., lines from the points a', b', c', &c., to v' being the edges of the pyramid in the elevation, and lines from a, b, c, &c., to v being the plans of these edges.

Examples.

1. A cube of 1″ 5 edge has its base horizontal, and at 1″ above the horizontal plane. Draw its projections, first, when one face is parallel to the vertical plane; secondly, when one face is inclined at 25° to the vertical plane.

2. A cube of 2″ edge rests upon the horizontal plane, and one face is inclined to the vertical plane at an angle of 30°; upon the centre of the cube stands a square pyramid 2″·5 high; each edge of the base is 1″·6 long, and is parallel to one edge of the cube. Draw plan and elevation.



O
A
B
C
D
E
a'
b'
c'
d'
e'
T'
T
v'
o
a
b
c
d
e
v

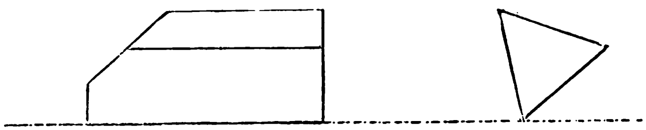
3. Draw the plan and elevation of an octagonal pyramid, diameter of the base 2″ and height 2″·5.

4. Draw the plan and side elevation of a square prism, edge of base 1″·25, and height 2″·25, when the bases are vertical, i.e. the axis is horizontal.

5. A circular slab of 3″·25 diameter, and 1″·5 thick, lies upon the horizontal plane, a cone 1″·5 diameter and 2″·12 high stands upon it, the axis of the cone passes through a point 0″·75 from the centre of the slab. The top of the cone is cut off by a plane inclined at 54° to the horizontal plane, and passing through the axis at a point 1″·12 above the base. Draw plan and elevation of the slab and truncated cone.

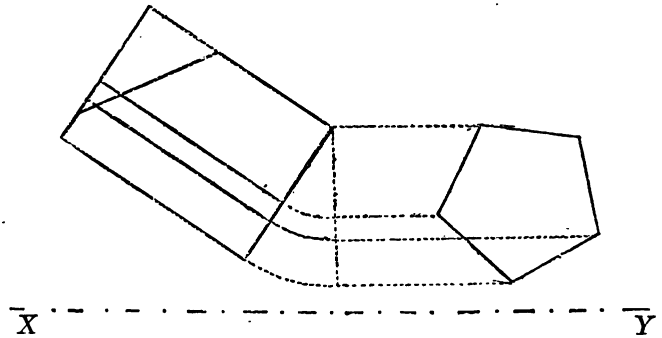
6. Draw plan and elevation of an octahedron, each edge 1″, the axis horizontal and parallel to the vertical plane.

7. A rectangular block, length 3″·5, width 2″·75, height 1″·5, lies on the horizontal plane; a cylinder 3″·5 long and 1″·75 diameter lies horizontally on the block, the axis of the cylinder being over the centre line of the block. Draw the plan and elevation of the whole, first, when the long sides of the block and axis of the cylinder are parallel to the V.P.; secondly, when they are inclined to the V.P. at 35°.



8. The above figures are the side and end elevation of a prism which has one corner cut off by a plane at right angles to the V.P. Draw the plan, and also draw the section made by the plane in its true form.

9. The figures below represent a side and end elevation of a prism, one corner of which is cut off by a plane at right angles to the V.P. Draw the plan, and show the true form of the section.



The student who has carefully mastered the explanations and problems in the above sections, and has worked out the examples given at the end of each should now be prepared to pass without difficulty the Examination in Practical Geometry) so far as the Geometry of Solids is concerned) for a Second Grade Art Certificate ; it must not be forgotten, however, that neatness and accuracy of drawing are expected as well as a correct solution of the questions given.

The candidate for the Elementary Science Examination in Practical Geometry must go through the remainder of this little work, so as to understand thoroughly the method of describing planes by their traces, and placing lines in given positions in any plane, and also be prepared to make projections of solids of somewhat less simple form.

CHAPTER VI.

FURTHER PROBLEMS ON PLANES AND LINES.

PROBLEM XXVII.

REPRESENT by their traces the following planes :—

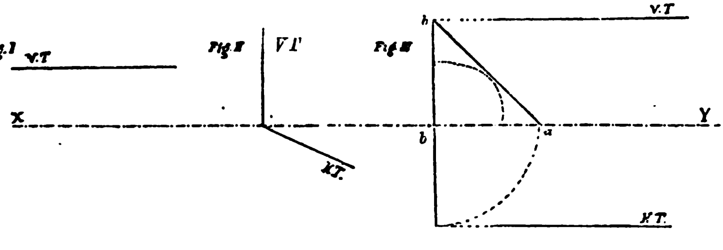
i. A plane which is horizontal and 0″·6 above the H.P.

ii. A plane which is vertical and inclined at 25° to the V.P. of projection.

iii. A plane inclined at 45° to both planes of projection and ·75 from the ground-line.

iv. A plane inclined at 30° to the H.P. and at 60° to the V.P.

v. A plane inclined at 30° to the H.P. and at 90° to the V.P.

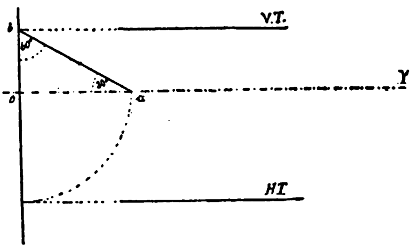


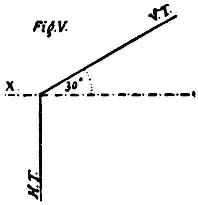
i. A horizontal plane can have no horizontal trace. Draw a line 1″·2 above XY and parallel to it; this will be the V.T.

ii. As the plane is vertical its V.T. will be a line perpendicular to XY, and as the plane is inclined at 25° to the V.P. its trace upon the H.P. will be inclined at 25° to XY.

iii. This plane being equally inclined to both planes of projection, the traces will be equidistant from XY and parallel to it; imagine the planes of projection to be turned so as to be seen sideways as *oa* and *ob*, then the inclined plane will appear as the line *ab* drawn diagonally and touching an arc struck from *o* with a radius of 1″·5. Then *oa* and *ob* (which in this case are equal) show the distance of the H.T. below and the V.T. above the ground-line.

Fig. IV





iv. This is similar to the above except that instead of drawing a diagonal inclined at 45° to both planes, a line must be drawn inclined at 60° to the line which represents the V.P. and at 30° to the H.P.; the distances *oa* and *ob* then give as before the distances of the traces below and above XY.

v. This plane being at right angles to the V.P. the H.T. is perpendicular to XY, and the V.T. makes an angle of 30° with XY.

Problem XXVIII.

Draw a line inclined at 20°—

i. In a plane which is inclined at 30° to the H.P. and perpendicular to the V.P.

ii. In a plane inclined at 30° to the H.P. and not perpendicular to the V.P.

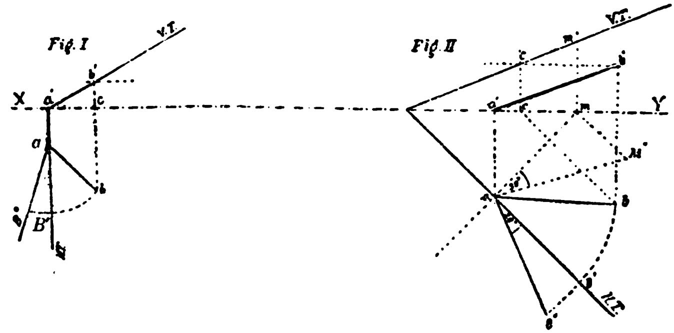
iii. In any plane given by its traces and inclined at not less than 20°.

i. Draw H.T. and V.T., Fig. I., the traces of a plane at right angles to the V.P. but inclined at 30° to the H.P. Take in the H.T. any point *a* from which to draw the line required; now if the line were taken in H.T. it would be horizontal, and if taken at right angles to H.T. it would be inclined like the plane at 30°; a line inclined at 20° must therefore lie somewhere between the two.

Set off at *a* (or, if preferred, a separate diagram may be drawn) an angle of 20°; take on H.T. any length aB′ and draw the perpendicular B′B″; then if a line equal to aB″ be inclined at 20°, and have the extremity *a* in the H.P., it is evident that its plan will be equal to aB′, and that the vertical height of the extremity B above the H.P. will be equal to B′B″.

From *a*, describe an arc with radius aB′; then if one extremity of the plan is at *a* the other extremity must lie in this arc. Also draw a line parallel to XY, and at a distance above it equal to B′B″, and meeting the V.T. in *b′*, and through *b′* draw a line at right angles to XY to meet it in *c*, and through *c* draw a line parallel to the H.T. (which in this case is the same thing as continuing the line *b′c*) to meet the arc in *b*. Join *ab*.

Now as the H.T. of any plane and all lines drawn parallel to the H.T. are horizontal lines, every point in any such line is at the same height above the H.P., therefore all the points in the line CB, of which *cb* is the plan, are at a height above the H.P. equal to *cb′* which was taken equal to B′B″. Therefore, as the base *ab* is equal to aB′, and the perpendicular at *b* is equal to the height B′B″, the hypotenuse aB of the right-angled triangle standing on *ab* would be equal to aB″ and the angle at *a* must be 20°; therefore *ab* is the plan of a line lying in the plane and inclined at 20°, and *a′b′* is its elevation. Four such lines may be drawn, two to the right of the H.T. *above* the H.P., and two to the left *below* it.



ii. Assume a H.T. not at right angles to XY, Fig. II., from any point in it a draw am at right angles to the H.T. and meeting XY in m. From a draw aM′ making an angle of 30° with am (or a separate figure may be drawn if convenient) and at m set up mM′ at right angles to am, then, as above, if the plane is inclined at 30° am is the base of a right-angled triangle which is supposed to stand upon it, mM′ the height, and aM′ the hypotenuse is the actual length of the line aM lying in the plane.

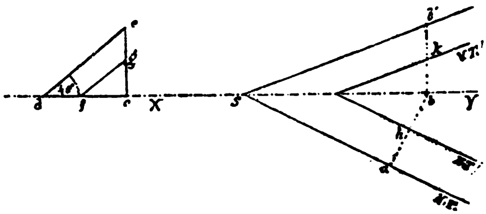
From m set up mm' equal to mM′, and through m' draw the V.T., meeting the H.T. in XY. We have now the traces of a plane inclined at 30°; for further explanation see next problem.

The remainder of the problem is exactly as in case i. above, except that after the height $c'c$ is taken equal to the height B′B″, the line through c which is drawn parallel to the H.T., being the plan of a horizontal line lying in the plane, is not at right angles to XY as it was in that instance. The plan ab being fixed, the elevations of A and B are found by remembering that as A is a point in the H.P. its elevation is in XY, and that B is at the same height above the H.P. as B″ or c'.

iii. This is precisely the same as the case ii. above, except that, both traces being given, it is not necessary to find the V.T.

PROBLEM XXIX.

Draw the V.T. of a plane inclined at 40°, of which the H.T. is given; also show a second plane parallel to and 0″·5 below it.



Let H.T. be the given horizontal trace, meeting XY in S, from any point in it *a* draw *ab* at right angles to the H.T. and meeting XY in *b*; then construct (either on *ab* or as above in a separate figure) a right-angled triangle with the base *dc* equal to *ab*, and the angle at *d* equal to 40°; take the height of this triangle *ce* and at *b* draw *bb′* perpendicular to XY and equal to *ce*; through S and *b′* draw a line, which is the V.T. required as in case ii above.

If a vertical plane (know as an auxiliary plane) be supposed to stand on the line *ab*, cutting the H.T. at right angles, it will meet the V.P. of projection (which, it will be remembered, stands upright upon the line XY) in the line *bb′*, and thus when seen sideways there will be a triangle of which the base *ab* is equal to *cd*, and the height *bb′* equal to *ce*, therefore the angle at *a* between the inclined plane and the H.P. must be equal to the angle *cde*, or 40°. If this is not quite clear to the student he should place two cards to represent the H.P. and V.P., draw on the former the H.T. and the line *ab*, and on the latter the line *bb′*, cut out of paper a triangle *cde* and fit it in with the base *cd* coinciding with *ab*. Then

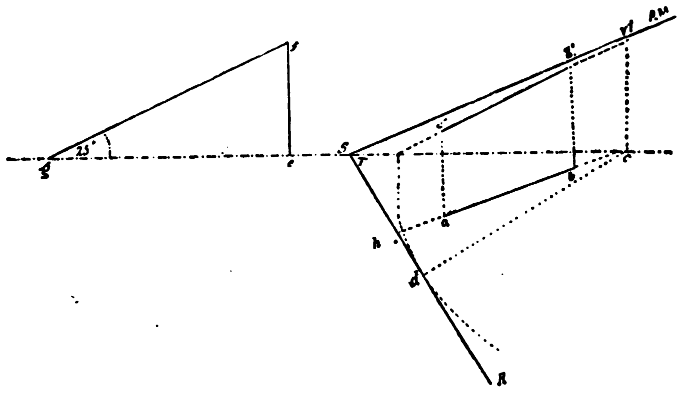
if he cuts another triangle with one side equal to S*a*, another equal to S*b*′, and the third equal to *de*, the hypotenuse of the triangle which stands upon *ab*, he will see clearly that S*b*′ is the V.T. of a plane inclined at the angle *cde*.

When one plane is parallel to another, their respective horizontal and vertical traces are parallel to each other.

In the triangle *cde* draw a line *fg* parallel to *de* (which represents the plane inclined at 40°), and at ·5″ from it; on *ba* take *bh* equal to *cf*, and on *bb*′ take *bk* equal to *cg*, and through *h* and *k* draw lines H.T′. and V.T′. parallel to the H.T. and V.T. These lines will meet in the ground-line, and be the traces of the plane required parallel to the first plane.

PROBLEM XXX.

Draw the traces of a plane which shall contain the line AB and be inclined at 25°.



It is evident that if a line be drawn from any point in the H.T. of a plane to any point in the V.T. this line will be in the plane, because the line joining any two points in a plane must lie entirely in the plane; therefore the converse is true, and if we find the horizontal and vertical traces of the line AB, and through them draw any two lines SM and SN meeting in

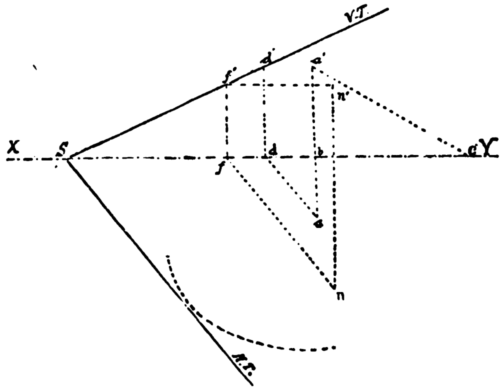
the ground-line in the point S, these will be the traces of a plane which contains the line AB; it is required, however, that the plane shall be inclined to the H.P. at 25°.

From the *v.t.* drop a perpendicular to the ground-line, meeting it in *c*, and construct a right-angled triangle with this line *c v.t.* as height *ef*, and the opposite angle *egf* equal to 25°; from *c* with radius *eg* describe a circle, and through the *h.t.* of the given line draw a line touching this circle in *d* and meeting the ground-line in S; from S draw another line through the *v.t.*; these will be the traces of the plane required.

It will be seen, as in the last problem, that if a line DC lies in the plane in a direction at right angles to the H.T., as the plan of this line or base of the imaginary triangle is *dc*, while the height of the end C is equal to *cv.t.*, then the slope of the plane must be 25°; and as the H.T. of it passes through the *h.t.* of the line, and the V.T. through the *v.t.* of the line, the plane contains the line AB.

PROBLEM XXXI.

Show by its traces a plane, not at right angles to the V.P., inclined at 30° and containing the point A, of which the projections are given; find also the elevation of the point N which lies in the plane, and of which the plan *n* is given.



It is evident that through a given point any number of planes can be drawn inclined at 30° to the H.P., in fact the point A may be supposed to be at the apex of a cone the surface of which, sloping from the apex regularly in every direction, meets the H.P. at an angle of 30°, then any plane which lies against the surface of the cone will pass through the point A, and fulfil the given conditions.

Construct a right-angled triangle with $a'b$ (the height of the point A above the H.P.) for its height, and with the opposite angle $a'cb$ equal to 30°, then this triangle represents half a section of the cone made by a vertical plane passing through the axis, and if from the plan a with radius bc we describe a circle, this will represent the base of the imaginary cone. Draw any line tangent to this circle for the H.T. of the plane required, and let it meet the V.P. in S.

Through the plan a draw a line ad parallel to the H.T.; this will be the plan of a horizontal line, and the point of which d is the plan will be at the same height above the H.P. as the point A of which a is the plan; therefore, if dd' is made equal to ba', d' is the elevation or the point D itself, and a line drawn through S and d' will be the V.T. of the plane required, for since it passes through the point d' it will also pass through the point A.

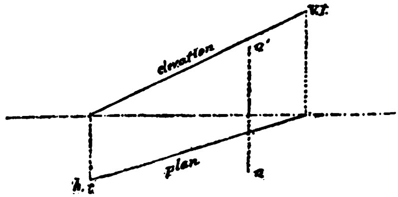
To find the elevation of N this process is reversed; draw through the plan n a line parallel to the H.T. and meeting XY in f, and draw ff' to meet the V.T. in f', then as ff' is the height of a point f' or F, it is also the height of N above the H.P., and this same height above XY must be set off above XY on a line drawn at right angles to it through the plan n.

Problem XXXII.

A line is given by its traces; does it pass through the point A, of which the projections are given?

If the point A is in the line of which the traces are given its projections must be respectively in the projections of the line.

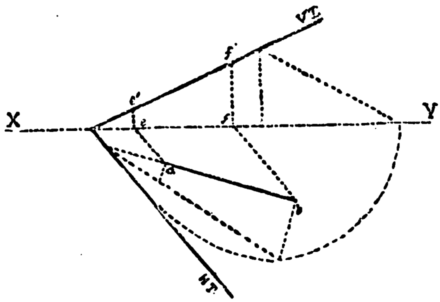
Draw these; it will be seen that a is not in the plan, nor is a' in



the elevation of the line, therefore the point A is not in the given line.

Problem XXXIII.

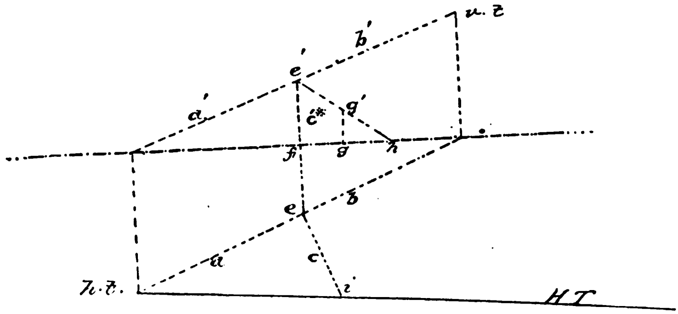
The line of which $a\ b$ is the plan lies in the plane of which the V.T. is given and which is inclined at 30°; what is the true length and inclination of AB?



The H.T. must first be found by applying the same principle as in Problems XXIX., XXX., and XXXI. The heights of A and B above the ground-line are then found by drawing through a and b lines parallel to the H.T. to meet XY in e, f, and drawing through e and f perpendiculars to meet the V.T. in e' and f'. These heights being found, the true length and inclination of AB are found as in Problem VIII., Chapter III.

Problem XXXIV.

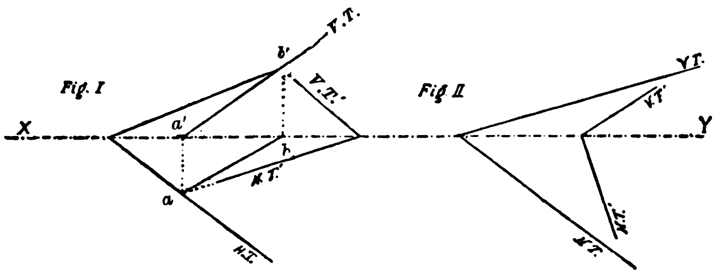
Find the traces of the plane containing the three given points A, B, and C.



The first part of this problem is the same as Problem XXX. By joining ab and $a'b'$ we get the projections of a line AB, and when the traces of this are found any number of planes can be drawn which would contain the line; it is only necessary to draw the one which will contain the point C. From the plan c draw a perpendicular to meet the plan ab (produced if necessary) in e, and find the height of this point in ab above the H.P., which will be fe'; make fg equal to ec and a perpendicular gg' equal to the height of C above the H.P., join $e'g'$ and produce this line to complete the triangle in h; then as fg is the length of base corresponding to the difference of height between C and E, gh is the length of base corresponding to the difference between C and the H.P.; that is to say, if ec is produced to i, making ci equal to gh, a plane cutting the H.P. at i and passing through the point C will also pass through the point E. Through the $h.t.$ of the line ab and i draw a line; then this is the H.T. of the plane required, for i is one point in this trace and the $h.t.$ of the line is necessarily another. The V.T. of the plane can be found as in preceding problems.

PROBLEM XXXV.

Find the projections of the line of intersection of the two planes which are given by their traces.



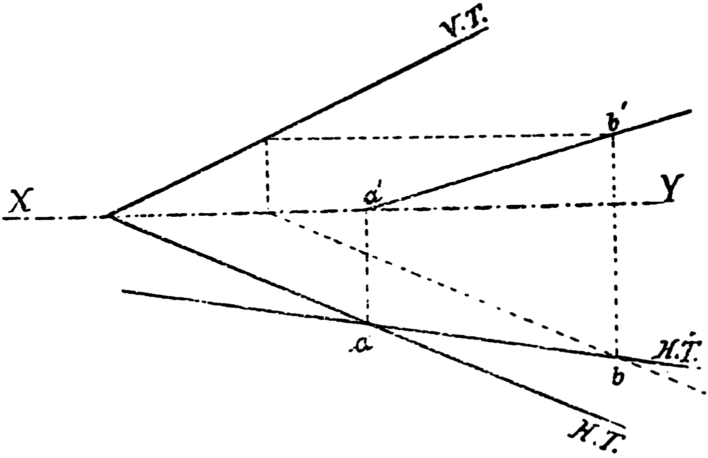
The point where the horizontal traces of two planes intersect is evidently one point in the line in which the planes intersect each other, and similarly the point in which the vertical traces intersect is another point, and as the projections of a straight line are lines joining the projections of the extremities of (or of any other points in) that line, it is only necessary to find the projections of the points in which the traces intersect and to join these. Let H.T'. and V.T'. meet H.T. and V.T. in points *a* and *b'* respectively, find the other projections *a'* and *b*, and join *ab*, *a'b'*; these are the projections required. The planes may slope either in opposite directions as in Fig. I., or in the same direction but at different angles as in Fig. II.

PROBLEM XXXVI.

H.T. and V.T. are the traces of a plane and H.T'. is the horizontal trace of a vertical plane. Show their intersection.

As the second plane is vertical, the horizontal trace H.T'. is itself the plan of the line of intersection: it is required to find the elevation of it. The point *a* where this line meets the H.T.

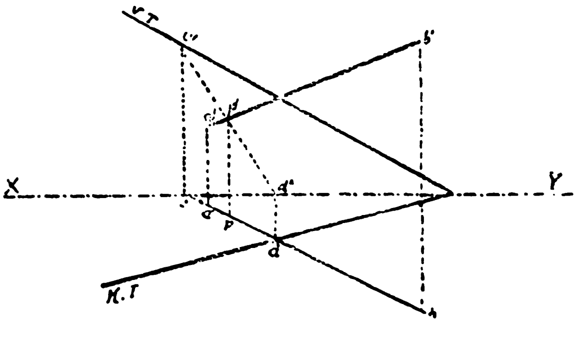
lies in the H.P., so a' in XY is its elevation. Find the elevation of a second point b by drawing through it a line parallel to the H.T. to meet XY, and another at right angles to



XY, which will give the height of b'; then $a'b'$ is the elevation required. It may be continued beyond a' as a line below the H.P. and also beyond b'.

PROBLEM XXXVII.

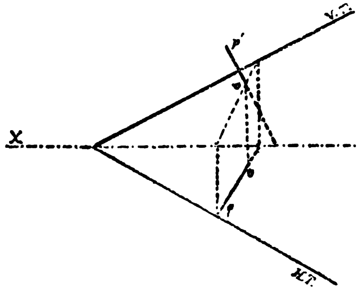
Find the intersection of the line AB given by its projections with the given plane.



Imagine a vertical plane to stand upon the plan *ab* and pass through the line itself AB, this plane (produced) will meet the V.P. in *cc'*, and AB will meet the given plane somewhere in an imaginary diagonal line drawn from *c'* to *d*, the line in which the auxiliary V.P. meets the inclined plane; the elevation of this line is *c'd'*, and the intersection of this line with the elevation *a'b'* shows the height above the H.P. at which the intersection takes place, thus the elevation of the point *p'* is obtained, and from this the plan *p*.

Problem XXXVIII.

From the point P which is given by its projections draw a line perpendicular to the given plane.



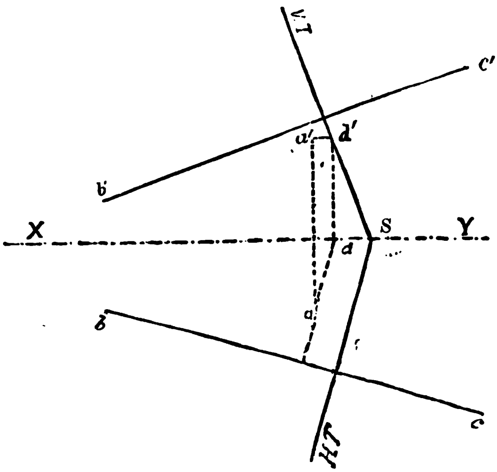
When a line is perpendicular to a plane the projections of the line are perpendicular respectively to the traces of the plane. The student will see this easily by the help of a model. Imagine a vertical plane to pass through the line which is perpendicular to an oblique plane, this plane must be at right angles to the oblique plane as it contains a line at right angles to it, and therefore the line in which this vertical plane of projection cuts the oblique plane is at right angles to the horizontal line in which the oblique plane meets the H.P., and which is called the horizontal trace. But the line in which this vertical plane of projection meets the oblique plane is vertically over the line in which it meets the H.P., that is to say the *plan of the perpendicular line*, and in the drawing the lines coincide; therefore

the plan of a line perpendicular to an oblique plane is at right angles to the H.T. of the plane, and similarly the elevation is at right angles to the V.T.

All that is necessary therefore is to draw through p and p' lines at right angles respectively to the H.T. and the V.T. of the given plane, and by Problem XXXVII. the plan and elevation, o and o', can be found of the point in which the perpendicular PO meets the plane.

Problem XXXIX.

Through the point A which is given by its projections draw a plane perpendicular to the line BC.

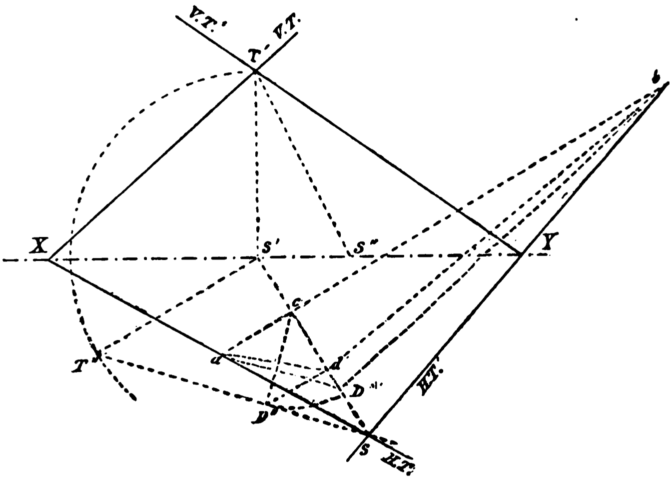


From the last problem it is evident that the traces of the plane will be at right angles to the projections of the line; through the plan a draw a line ad at right angles to the plan bc and meeting XY in d, then ad is the plan of a line drawn through the point A parallel to the H.T. of the plane which is to be drawn, therefore it is the plan of a horizontal line lying in the required plane. Draw $a'd'$ the elevation of the same line, then d' is the point in which this line in the required

plane will meet the V.P., consequently it must be a point in the V.T.; through d' draw a line at right angles to $b'c'$ and meeting XY in S; this will be the V.T. of the plane required, and through S the H.T. can be drawn at right angles to bc.

PROBLEM XL.

Find the angle between the two planes which are given by their traces.



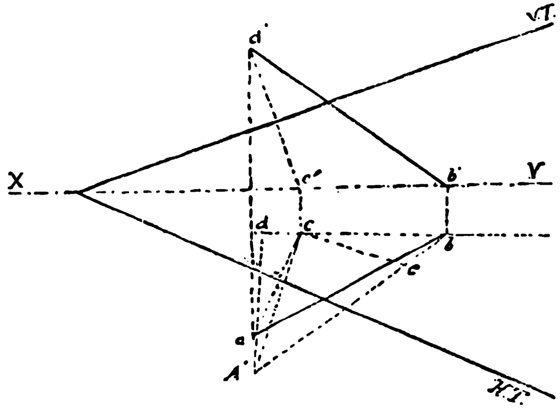
Let H.T., V.T., and H.T'., V.T'., be the traces of the two planes which intersect; then the line of their intersection from S to T is shown by its plan SS' and its elevation S'T'. Suppose a third plane perpendicular to this line of intersection to meet it in a point D (which cannot be at once shown in the drawing), and to meet the H.P. in the line ab, then we have a triangle of which the base is ab, the two sides are the lines in which this auxiliary plane cuts the two given planes, and the angle at D is the angle between these two planes, which has to be found. Now imagine the triangle between the points S, S' and T' to be laid down on its side, that is to say revolved upon the base SS' until it comes into the H.P.; it will then

appear as ST″S′, in which S′T″ is equal to S′T′, and the angle SS′T″ is a right angle, and if CD″ is drawn at right angles to T″S, this will represent the line in which the triangle when standing upright upon SS′ meets the auxiliary plane which cuts the line between S and T in the point D. When the triangle T″S′S is revolved back again until it stands vertically upon SS′, the point *d* is the plan of the point D; it remains to find the true form of the triangle of which *adb* is the plan.

Suppose this triangle *a*D*b* to be revolved forwards upon the line *ab* until it comes into the H.P.; then the vertex D will move over the line S′S until it comes into the position D‴, the distance of D‴ being the same as that of D″ from the point *c*, about which it revolves. Join D‴ to the points *a* and *b*; then *a*D‴*b* is the angle between the two planes required.

Problem XLI.

Find the angle between the line which is given by its projections and the given plane.

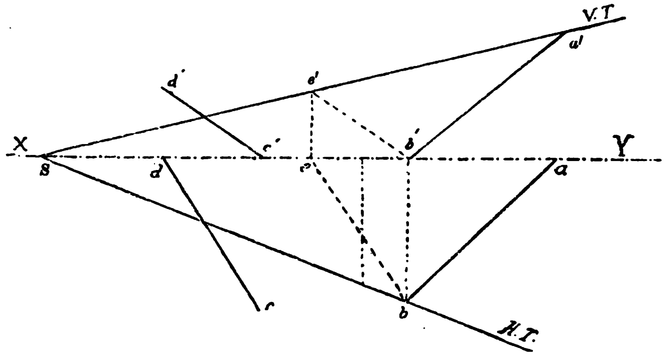


If the line does not meet the H.P., produce it till it does, and let *ab* meet the H.P. in *b*. From any point in this line, as *a*, draw a perpendicular to the plane; *ac* will be the plan and *a′c′* the elevation of this perpendicular, and *c* is the point in which

it meets the H.P.; join *bc*, and produce it to *d*, making *ad* perpendicular to *bd*. We have now a triangle, of which the point A at a certain height above its plan *a* is the apex, and lines from A to *d* and A to *b* are the sides, and when this triangle is revolved upon the line *bd*, the point A will come into the H.P. at A′, which is found by drawing a right-angled triangle, not shown in figure, with base *ad* and height equal to the height of *a′* above XY; the hypotenuse is evidently the actual distance of the point A from *d*, and gives *d*A′. Then the angle *c*A′*b* is the angle between the given line and a line perpendicular to the plane. Through *c* draw *ce* at right angles to A′*c*; then the angle A′*ec* is the angle between the given line and the plane.

Problem XLII.

Through the given line AB draw a plane which shall be parallel to the line CD.



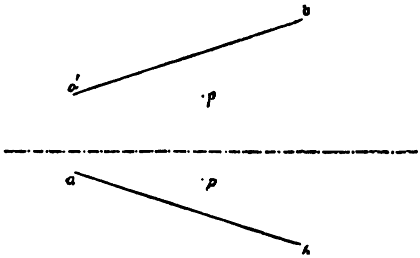
If the given line AB does not meet the two planes of projection, produce the plan and elevation to meet them respectively in *a* and *b′*, through *b* and *b′* draw the projections of a line BE parallel to CD (that is, draw *be* parallel to *cd*, and *b′e′* parallel to *c′d′* and meeting the V.P. in *e′*). Then a line drawn through *a′* and *e′* to meet the ground-line in S is the vertical trace of a plane which contains the lines AB and CD, and S*b* is the H.T.;

and because it contains a line parallel to CD the plane is parallel to this line.

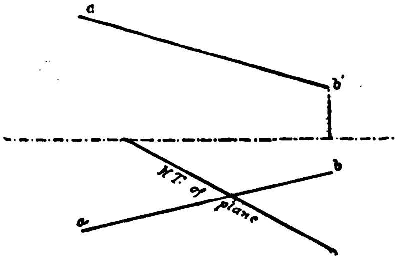
EXAMPLES ON LINES AND PLANES.

1. Represent, first, a vertical plane inclined at 45° to the V.P. of projection; secondly, a horizontal line lying in a plane whose traces both make angles of 50° with the ground-line.

2. Determine the horizontal and vertical traces of a plane containing the line AB and inclined at 50°.



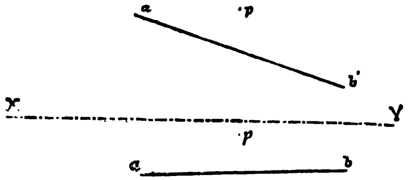
3. Find the intersection of the line AB with a plane inclined at 30°, of which the H.T. is given.



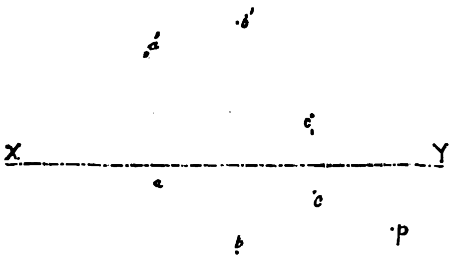
4. Draw the plan of an equilateral triangle of 2″ side lying in a plane which is inclined at 35°, one side of the triangle to be inclined at 25°.

5. Draw the projections of a line passing through P and parallel and equal to the given line. Join the extremities of

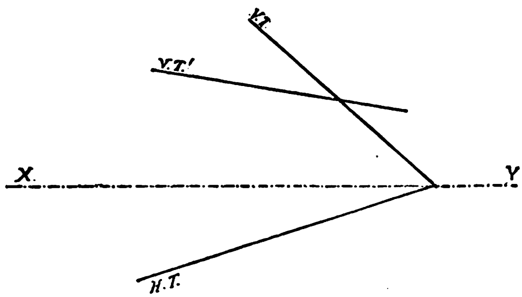
these two lines, and draw the actual form of the four-sided figure thus obtained.



6. A point P lies in the same plane as the points A, B, and C; find its elevation.

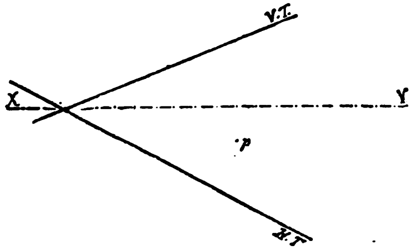


7. H.T. and V.T. are the traces of a plane, and V.T'. is the vertical trace of another plane perpendicular to the V.P.; determine the intersection of these two planes.

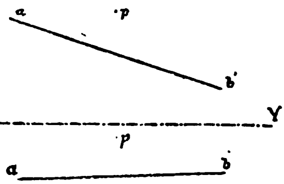


8. The point P lies in the plane of which the traces are given;

find the elevation of P, and draw through it a perpendicular to the given plane.

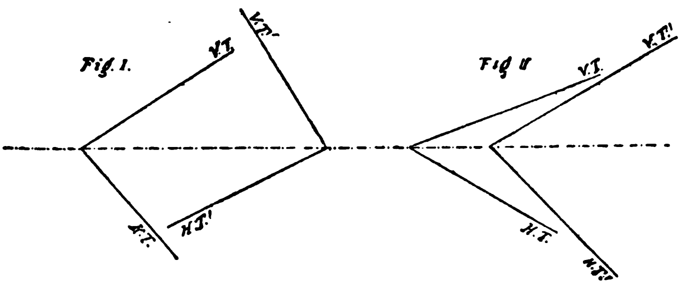


9. Determine the traces of a plane containing the line AB and the point P of which the projections are given.



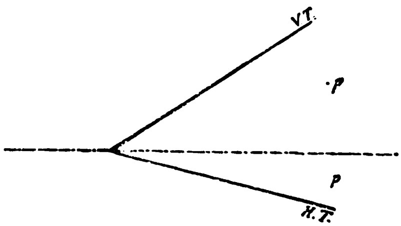
10. The horizontal and vertical traces of a plane are inclined to XY at 35° and 45° respectively; find the projections of a point in this plane which is 2″ from either plane of projection.

11. H.T. and V.T., H.T'. and V.T'., are the traces of two

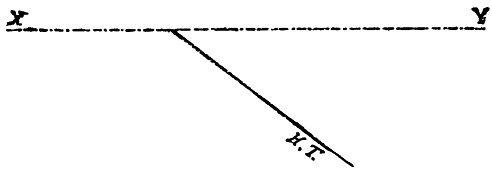


planes; find the projections of the line in which they intersect, and also the actual length and inclination of this line.

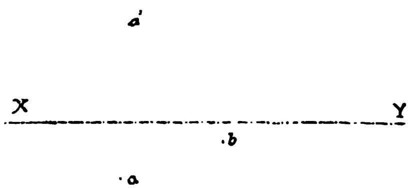
12. Through the point P draw a plane parallel to the given plane.



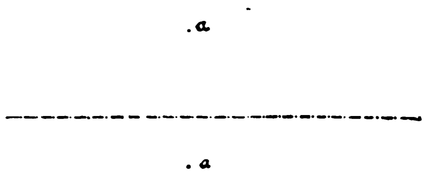
13. The H.T. is given of a plane which is inclined at 50°; draw its V.T. Also draw a second plane parallel to this and 0″·75 above it.



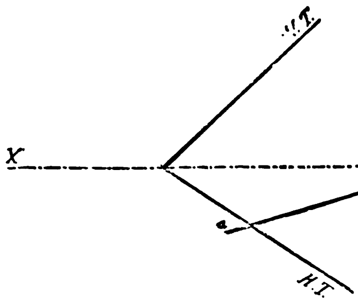
14. First, show by its traces a plane, not at right angles to either plane of projection, containing the point A, of which the projections are given; secondly, b is the plan of a point in it which is 2″·5 from A; find its elevation.



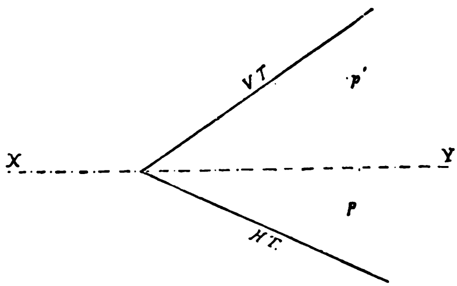
15. Show by its traces a plane inclined at 60° to the H.P. and containing the point A in figure on next page.



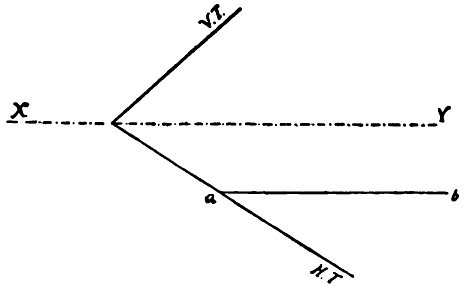
16. Find the inclination to the H.P. of the line AB, which is in the plane of which the traces are given.



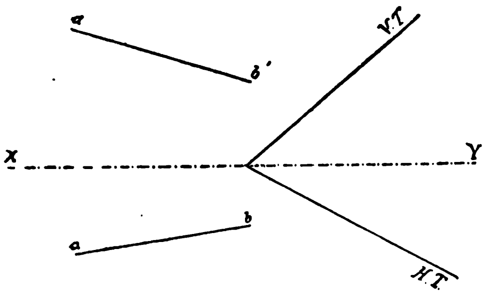
17. A plane is given by its traces, P is a point in it; from P draw two lines in the plane inclined to the H.P. at 25°, and determine the actual angle between these lines.



18. The line AB lies in the given plane; determine its elevation and real length.

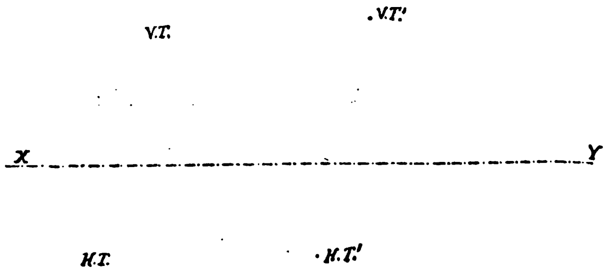


19. Find the intersection of the line AB with the given plane.



20. Find the intersection of two planes, the horizontal traces of which are parallel and 1″·25 apart, the planes being inclined in one direction, the lower one at 50° and the upper at 38°.

21. H.T. and V.T. are the traces of a line; H.T.′ and V.T.′ are the traces of a second line. Do the lines meet?



CHAPTER VII.

MISCELLANEOUS EXERCISES ON SOLIDS.

THE following exercises, which are taken from questions which have been already set in the Elementary Examination Papers of the Science and Art Department, contain no new principles or rules, but are only a development of those already explained in Chapter V. The student will understand how each is worked by carefully following the lines of construction, and it will be well to draw each on a larger scale.

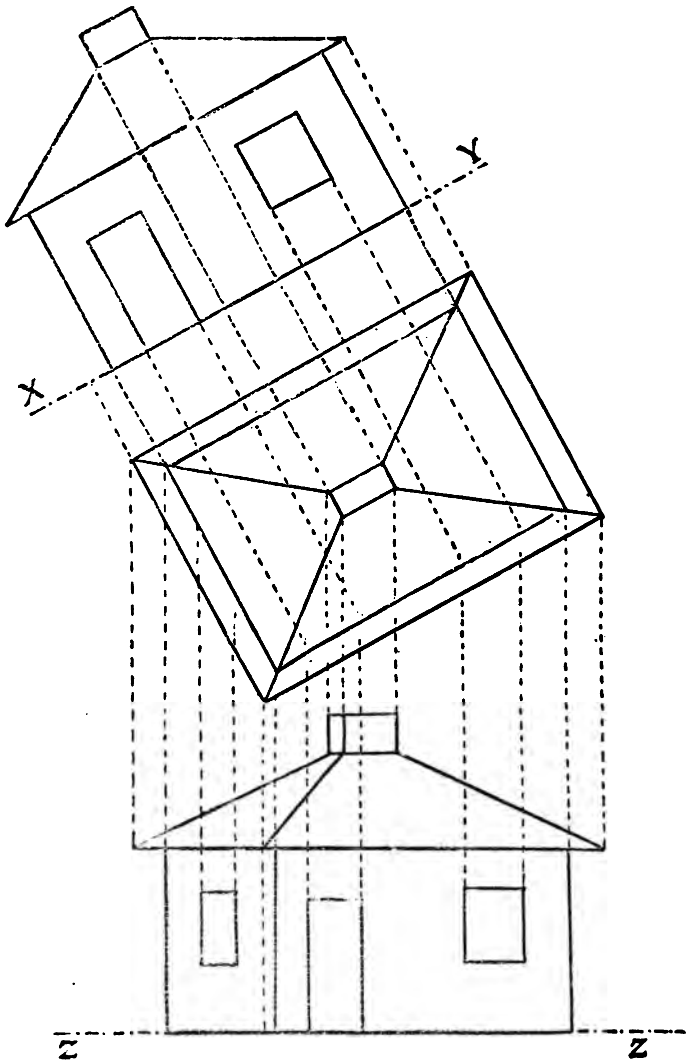
PROBLEM XLIII.

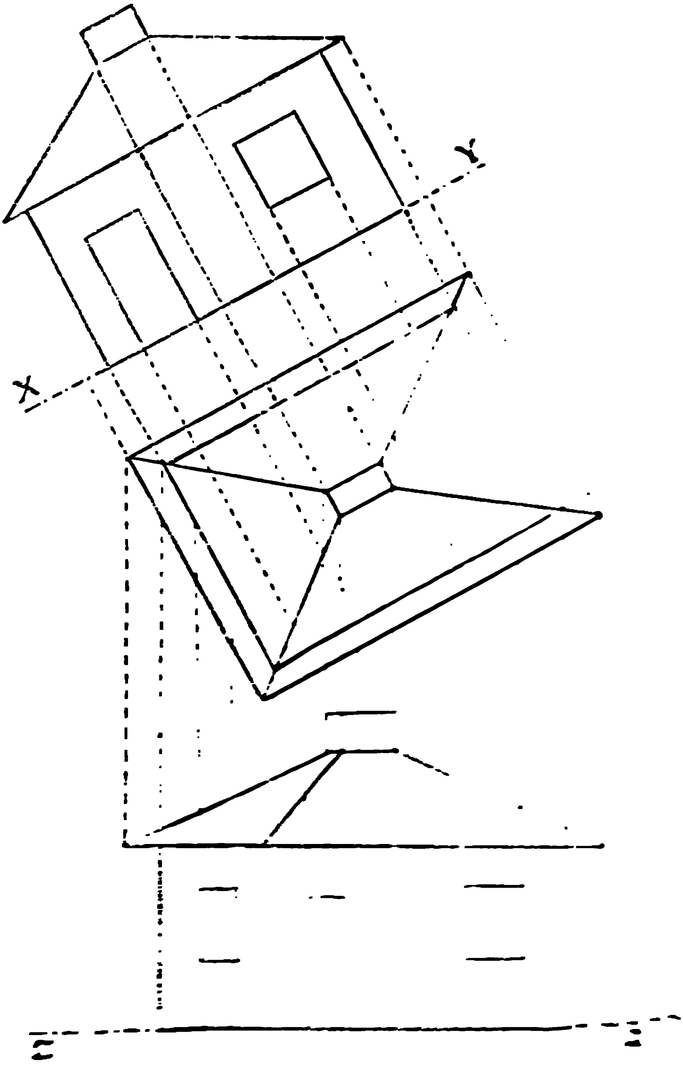
The plan and elevation of a cottage are given; draw an elevation of it on the line ZZ'. (NOTE. In the centre of the end wall there is a window of the same size and at the same height as the one shown in the front of the cottage.)

PROBLEM XLIV.

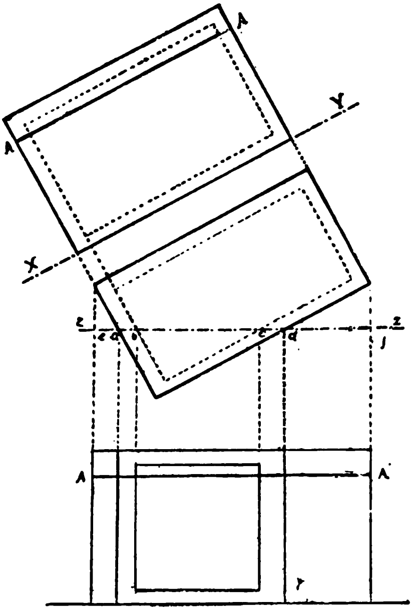
The figures show the plan and elevation of a box; the ends are $1\frac{1}{2}''$ thick; the remaining portions are $1''$ thick; draw a sectional elevation on the line ZZ'. Scale $\frac{1}{8}$ or $8'$ to $1'$.

In this case the lines which show the inside of the box at $1'$ from the top, bottom, and sides, and at $1\frac{1}{2}''$ from the ends, must first be drawn. To avoid confusing the figures the sectional elevation is made on another line parallel to ZZ'; in this the portion of the end of the box from e to a appears in elevation; from a to b is a section of the end piece of wood; from b to c is an elevation of the interior of the box with the top and bottom shown in section; from c to d is a section of the front of the box; and from d to f an elevation of the


Y
X
Z
Z


Y
X

remaining portion of the front of the box. The heights are all taken from the elevations above. AA and A′A′ show the line where the lid joins the box.



PROBLEM XLV.

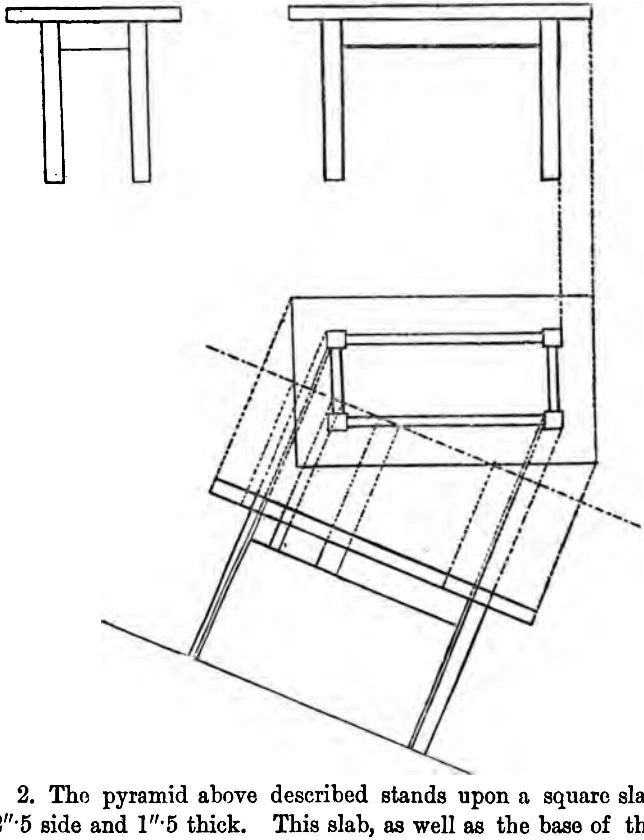
The side and end elevations of a table are given; the four pieces which form the frame are half as thick as the legs, and they are opposite to the centres of the latter. Draw a plan and make a sectional elevation on a line drawn across the plan, so as to cut off half one end and two-thirds of one side.

(*See Figure on opposite page.*)

MISCELLANEOUS EXAMPLES IN THE PROJECTION OF SOLIDS.

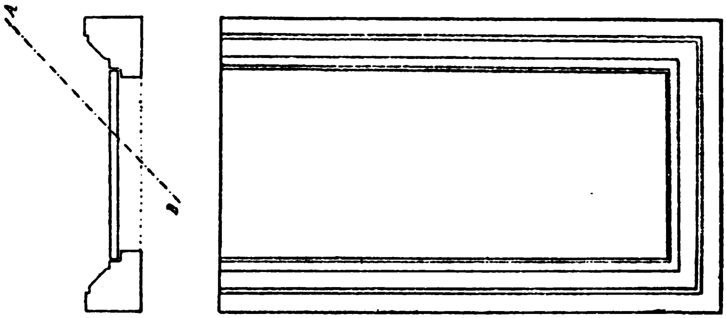
1. A hexagonal pyramid, height 3″, side of base 0″·75, stands upon the H.P. with one side inclined at 15° to the V.P.; draw its projections. A plane inclined to the H.P. at 25° passes

through the pyramid, and its H.T. is a line at right angles to the ground-line and 1″·25 from the centre of the plan; draw the plan of the section and also its true form.



2. The pyramid above described stands upon a square slab 2″·5 side and 1″·5 thick. This slab, as well as the base of the pyramid, is inclined to the H.P. at 20°; draw the plan, an elevation upon a V.P. parallel to the axis of the pyramid, and another upon a V.P. inclined to the first at 30°.

3. The plan and elevation are given of a door and door-frame; draw a sectional elevation on the line AB.



4. A wooden stool consists of a circular top 1′ in diameter and $1\frac{1}{2}''$ thick, with three legs joined to it at equal distances from each other and halfway between the centre of the top and its edge. The legs are square, $1\frac{1}{2}''$ thick, 2′ 6″ long and splay outwards, so that at the bottom the centre of each leg is under the edge of the top of the stool. Draw a plan and elevation : scale $\frac{1}{8}$ full size.

5. Fig. I. is the plan of two bricks lying one upon the other; each is $2\frac{3}{4}''$ thick. Draw an elevation of them on the line XY. The scale is $\frac{1}{4}$ full size.

Fig. I.

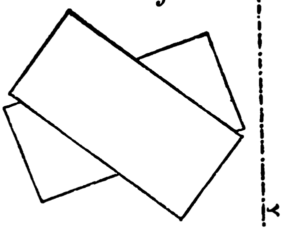
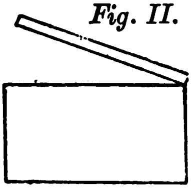


Fig. II.



6. Fig. II. is the end elevation of a box with the lid partly open; draw the plan, front elevation, and an elevation on a line inclined at 25° to the front line of the plan. The length of the box is $2\frac{1}{2}$ times the width.

GILBERT AND RIVINGTON, LIMITED, ST JOHN'S SQUARE, LONDON.